Immediate near-misses and resonant returns

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Apophis, near-misses and resonant returns

When Apophis was discovered, impact monitoring showed that, besides the possibility of a collision, there was also the possibility of a near-miss.

Nearly missing the Earth means a large orbital deflection, with a continuum of possible post-encounter orbits, among which there are some that can lead to a resonant return and to a collision.

The cross-sections of resonant returns are generally much smaller than the cross-section of the immediate collision; one reason why they are small is the divergence of nearby post-encounter trajectories, that accumulates with time, so that returns that are far in the future have very small cross-sections.

But, is it always so?

What about returns taking place after only 1, 2, 3 years?

Apophis, near-misses and resonant returns

We hereafter study the resonant returns of Apophis taking place within only three years after the very close encounter with the Earth of April 2029.

In particular, we consider the *b*-plane of that encounter.

The *b*-plane is the plane centred on the Earth and perpendicular to the geocentric unperturbed velocity $\vec{U}$ of the asteroid (Kizner 1961); $\vec{U}$ crosses the *b*-plane in the point of coordinates ξ, ζ (Valsecchi et al. 2003).

The coordinate ξ is the local minimum orbit intersection distance (MOID), while the coordinate ζ is related to the timing of the encounter.

The semimajor axis perturbation

Before the first encounter of the pair, the asteroid has an orbital semimajor axis given by:

$$a = 1/(1 - U^2 - 2U\cos\theta)$$

where U is the modulus of $\vec{U}$, and θ is the angle between $\vec{U}$ and the heliocentric velocity of the Earth.

The first encounter turns $\vec{U}$ into $\vec{U}'$, of the same modulus; θ is changed into θ', so that the post-first-encounter semimajor axis is a', given by:

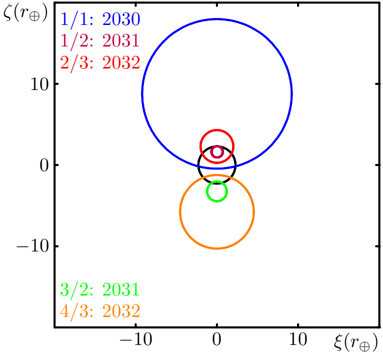
$$a' = 1/(1 - U^2 - 2U\cos\theta').$$

The *b*-plane circles

For a resonant return to take place, Apophis has to cross the *b*-plane in a point lying on a circle whose coordinates of the centre and radius are functions of the pre-encounter orbital elements, and of the post-encounter resonant period (Valsecchi et al. 2003).

The black circle shows the Earth cross-section, the coloured ones correspond to the mean motion resonances (MMRs):

2030, 1/1 MMR
2031, 1/2 MMR
2031, 3/2 MMR
2032, 2/3 MMR
2032, 4/3 MMR



The divergence of nearby trajectories

In a resonant return the first encounter puts the small body in an orbit such that it collides with the planet after h revolutions on the new orbit, while the planet makes k revolutions, with h, k integers.

Going from the b-plane of the first encounter to that of the second the orbit is "stretched": the separation along ζ increases due to differential perturbations of the post-encounter orbital period.

Thus, the cross-section of the planet on the b-plane of the second encounter has a pre-image on the b-plane of the first encounter that looks like a thin arclet, lying on, or close to, the b-plane circle corresponding to the h/k MMR (Valsecchi et al. 2003).

Chodas (1999) called "keyhole" the pre-image of the planet on the first b-plane of a resonant return.

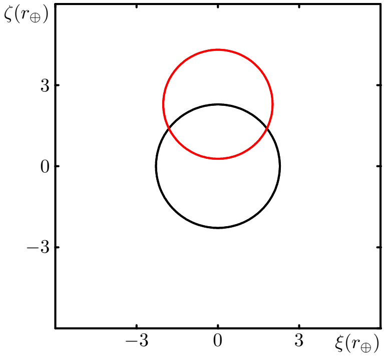
What a keyhole looks like

In the restricted circular 3-dimensional 3-body problem, we can reconstruct the shape in the 2029 *b*-plane of Apophis keyhole associated to the 2/3 MMR.

The black circle is the Earth cross-section.

The red circle shows the resonance condition.

The black dots correspond to *b*-plane crossings leading to collision with the Earth in 2032, computed numerically in the restricted problem.



The divergence of nearby trajectories

Let us denote with ζ'' the ζ coordinate on the second b-plane; we are interested in computing by how much ζ'' is stretched with respect to ζ between the first and the second encounter.

Valsecchi et al. (2003) give an expression for the partial derivative of ζ'' with respect to ζ:

$$
\begin{aligned}
\frac{\partial \zeta''}{\partial \zeta} &= h \cdot s \cdot \frac{\partial \cos\theta'}{\partial \zeta} + \mathcal{O}(1) \\
s &= \frac{2\pi[U\cos^2\theta' + \cos\theta'(1-U^2) - 3U]}{(1-U^2-2U\cos\theta')^{5/2}\sin\theta'} \\
\frac{\partial \cos\theta'}{\partial \zeta} &= \frac{2c\sin\theta}{\xi^2+\zeta^2+c^2} - \frac{4c\zeta(\zeta\sin\theta - c\cos\theta)}{(\xi^2+\zeta^2+c^2)^2},
\end{aligned}
$$

where θ' is the post-first-encounter value of θ and c is the geocentric b-plane distance leading to a deflection of $\vec{U}$ by 90°.

The divergence of nearby trajectories

In the partial derivative $\partial\zeta''/\partial\zeta$, the first term becomes quickly the dominant one.

It is the product of:

- the number of revolutions of the small body between the first and the second encounter, growing linearly with time;
- the function $s(U, \theta')$, that we discuss in a moment;
- the derivative $\partial \cos\theta'/\partial\zeta$, that can be large for very close encounters.

Digression: (410 777) 2009 FD keyholes

Before discussing the function s, let us see how the theoretical expression for $\partial\zeta''/\partial\zeta$ works in a real case.

In Spoto et al. (2014) are examined a number of Earth collisions at resonant returns originated by the close encounter of (410 777) 2009 FD keyholes in 2185.

These resonant returns are computed numerically both with the LoV method of Milani et al. (2005) and with a Monte Carlo method developed at JPL.

From Tables 2 and 3 of Spoto et al. (2014) we can gather the stretching increment between the 2185 encounter and each of the four resonant returns identified by both the LoV method and the Monte Carlo.

Digression: (410 777) 2009 FD keyholes

The Table below compares these stretching increments to the values of $\partial\zeta''/\partial\zeta$ computed by the theory.

The agreement is good with the LoV method, less good with the MC; however, the latter is affected by the limited resolution due to the relatively small number of initial conditions sampled.

Year	h/k	LoV	MC	Theory
2190	$4/5$	$1.6 \cdot 10^1$	$1.6 \cdot 10^1$	$1.6 \cdot 10^1$
2191	$5/6$	$2.0 \cdot 10^3$	$1.3 \cdot 10^3$	$2.1 \cdot 10^3$
2192	$6/7$	$6.0 \cdot 10^3$	$2.3 \cdot 10^3$	$6.6 \cdot 10^3$
2196	$9/11$	$1.2 \cdot 10^3$	$3.1 \cdot 10^3$	$1.4 \cdot 10^3$

The divergence of nearby trajectories

Let us now examine the expression for s:

$$s = \frac{2\pi[U\cos^2\theta' + \cos\theta'(1-U^2) - 3U]}{(1-U^2-2U\cos\theta')^{5/2}\sin\theta'}$$

The sign of s depends on the sign of the numerator; it is easy to see that $s=0$ for:

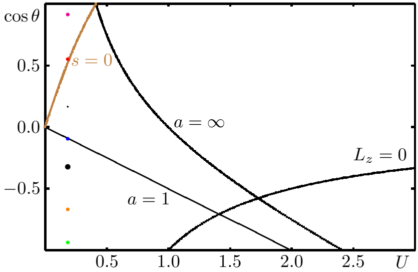
$$0 = U\cos^2\theta' + \cos\theta'(1-U^2) - 3U.$$

This condition defines a line in the plane U-$\cos\theta$.

The condition $s = 0$ and Apophis

In the U-$\cos\theta$ plane the big black dot shows the current position of Apophis.

The small dot shows the position of Apophis after 2029.



The coloured dots shows the MMRs we are discussing. Top to bottom: 1/2 (magenta), 2/3 (red), 1/1 (blue), 4/3 (orange), 3/2 (green).

The brown line is the condition $s = 0$.

We next compute the maximum keyhole widths for each of these resonances.

Apophis keyholes maximum widths

year	h/k	a' (au)	θ' ($^\circ$)	$\zeta(r_\oplus)$	s	width (km)
2031	$1/2$	1.5874	24.0	2.31	11.9	0.60
2032	$2/3$	1.3104	56.5	4.31	0.4	11.04
2030	$1/1$	1.0000	95.3	17.96	-4.0	23.98
2032	$4/3$	0.8255	131.8	-10.29	-7.8	1.41
2031	$3/2$	0.7631	159.3	-4.45	-17.6	0.38

What is noticeable in the Table is that keyhole widths are neither a simple function of the time interval between the two encounters (i.e. of k), nor of the closeness of the encounter (i.e. of ζ, through $\partial \cos\theta'/\partial\zeta$).

Particularly noteworthy is the role of s; we next explore different initial conditions for Apophis in 2029 that would highlight this aspect.

The keyhole associated to the 2/3 MMR

For the keyhole associated to the 2/3 MMR the value of s is close to 0.

Nevertheless, the orbit of the real Apophis, characterized by $U = 0.184, \theta = 108\overset{\circ}{.}7$, needs a very large deflection, by more than 52°, in order to be deflected into the 2/3 MMR.

The large deflection implies a very close encounter, closer than the one which will actually take place in 2029; anyway, the keyhole size can at most be of the order of 10 km.

What if, before the 2029 encounter, Apophis had the same geocentric velocity U, but a value of θ differing by just a few degrees from that corresponding to the 2/3 MMR?

A "what if" scenario

Let us suppose that Apophis were, before the 2029 encounter, in an orbit with $U = 0.184$ and $\theta = 60\overset{\circ}{.}5$, just 4° away from the 2/3 MMR.

In this case, the orbital semimajor axis would be 1.2739 au (the real Apophis now has $a = 0.9226$ au); the Table shows the corresponding theoretical maximum keyhole widths.

year	h/k	a' (au)	θ' ($^\circ$)	$\zeta(r_\oplus)$	s	width (km)
2031	1/2	1.5874	24.0	6.38	11.9	2.76
2032	2/3	1.3104	56.5	60.48	0.4	1870.17
2030	1/1	1.0000	95.3	−6.73	−4.0	3.65
2032	4/3	0.8255	131.8	−2.94	−7.8	0.17
2031	3/2	0.7631	159.3	−1.81	−17.6	0.12

Note that a not very deep encounter is needed to reach the 2/3 MMR, and that the maximum keyhole size is really very large.

The importance of s

On the other hand, if before 2029 Apophis had still $U = 0.184$ but $\theta = 127\overset{\circ}{.}8$, it would again be just 4° away from a MMR, this time the $4/3$. The orbital semimajor axis would be 0.8392 au, and the corresponding theoretical maximum keyhole widths would be those in the Table below.

year	h/k	a' (au)	θ' ($^\circ$)	$\zeta(r_\oplus)$	s	width (km)
2031	1/2	1.5874	24.0	1.65	11.9	0.44
2032	2/3	1.3104	56.5	2.94	0.4	6.28
2030	1/1	1.0000	95.3	7.22	−4.0	4.16
2032	4/3	0.8255	131.8	−60.26	−7.8	46.27
2031	3/2	0.7631	159.3	−7.47	−17.6	0.95

Now, even if for the $4/3$ MMR the encounter distance and the time between encounters are the same as previously for the $2/3$, the maximum keyhole size is much smaller, the difference being due to the value of s.

Summary

- Among the keyholes present on the *b*-plane of a near-miss there may well be some associated to "quick" resonant returns, i.e. returns taking place only a few years after the near-miss.
- In general, the size of keyholes associated to "quick" resonant returns is small.
- The analytic theory of close encounters shows that there are peculiar conditions under which a specific keyhole associated to a "quick" return may be large.
- To further assess the validity of the theory, the above conditions will be the object of numerical checks.

References

- Chodas, BAAS 31, 1117 (1999)
- Kizner, Planet. Space Sci. 7, 125 (1961)
- Milani, Chesley, Sansaturio, Tommei, Valsecchi, Icarus 173, 362 (2005)
- Spoto, Milani, Farnocchia, Chesley, Micheli, Valsecchi, Perna, Hainaut, A&A 572, A100 (2014)
- Valsecchi, Milani, Gronchi, Chesley, A&A 408, 1179 (2003)