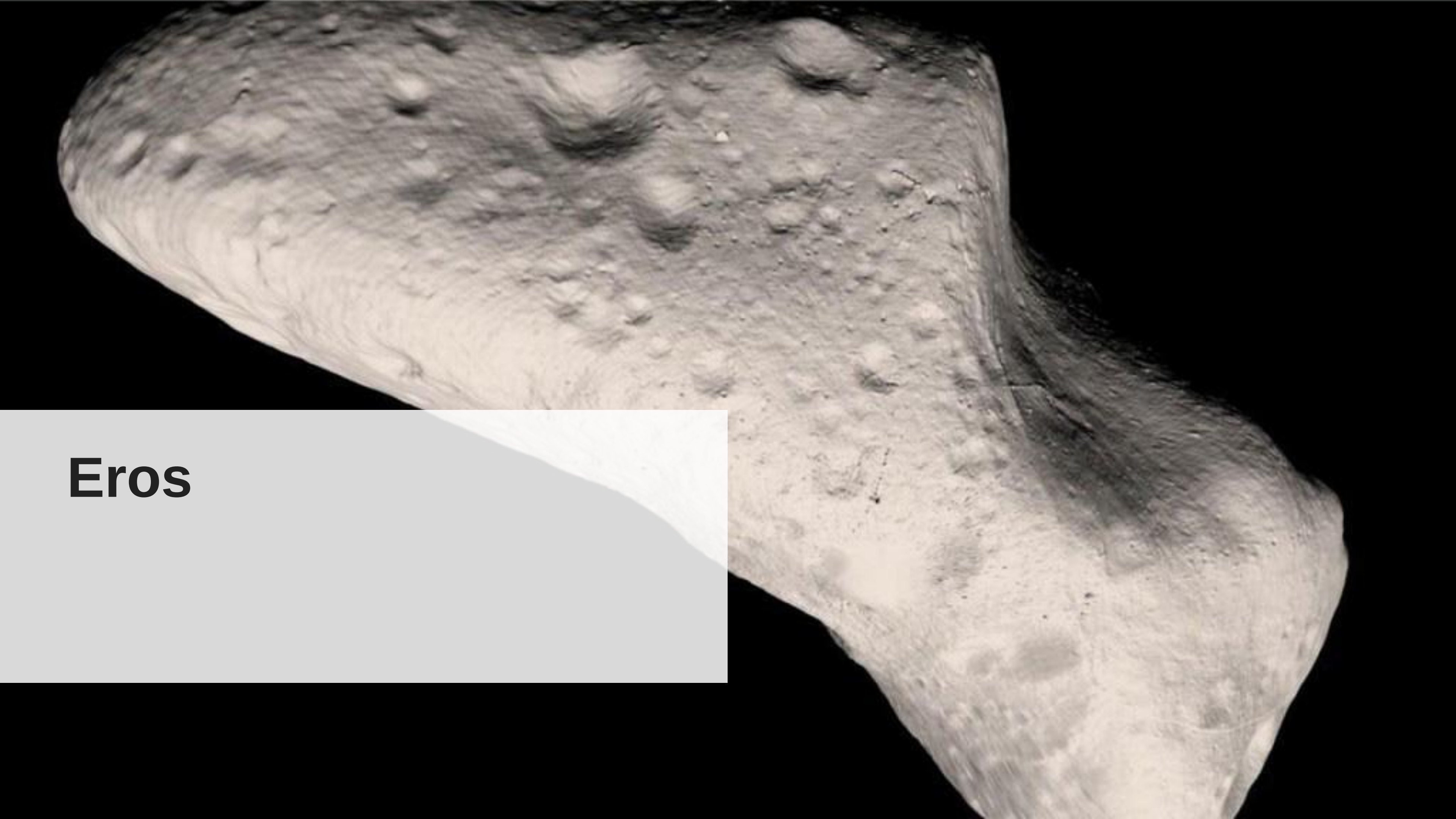
Neural Implicit Representation of irregular celestial bodies: GeodesyNETS, EcipseNETS and more

by Dario Izzo and Alexander Zoechbauer

Irregular bodies in the solar system

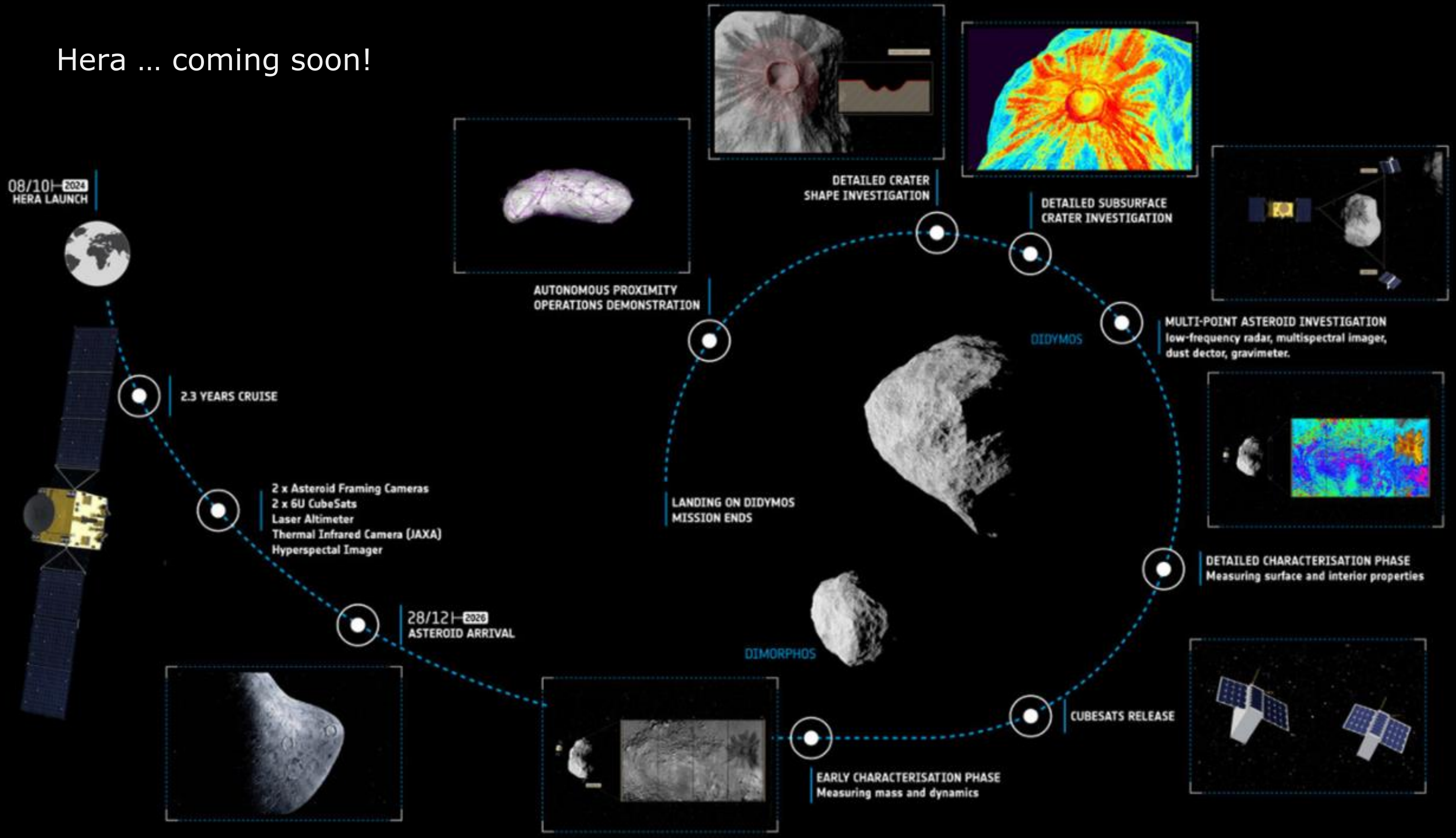
Eros



Itokawa

67P/Churyumov Gerasimenko

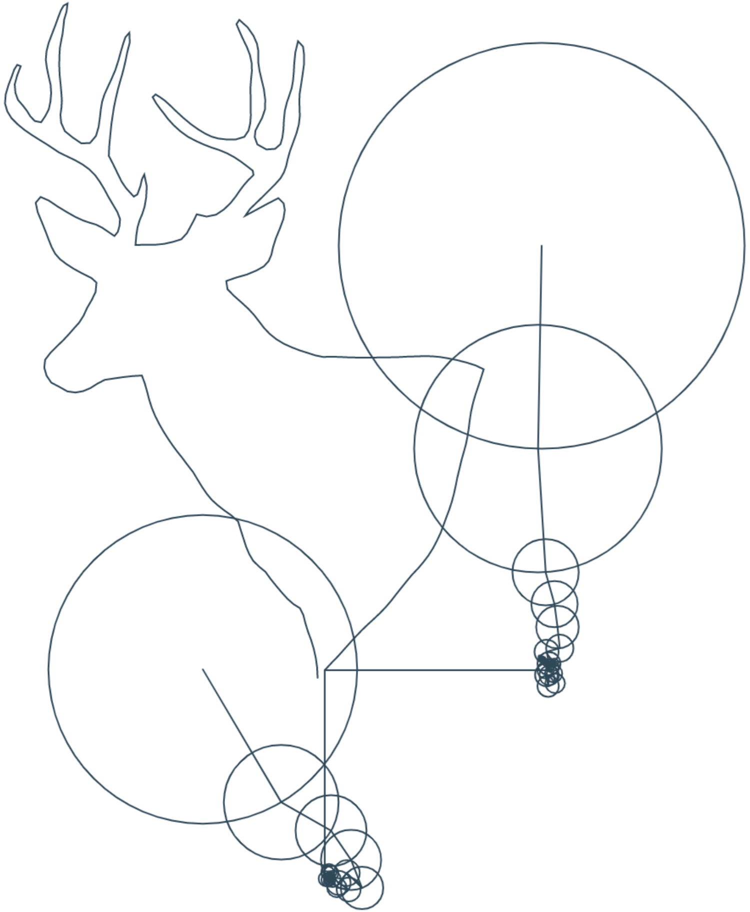
Hera … coming soon!



Implicit neural representations

“With four parameters I can fit an elephant, with five I can make him wiggle his trunk”

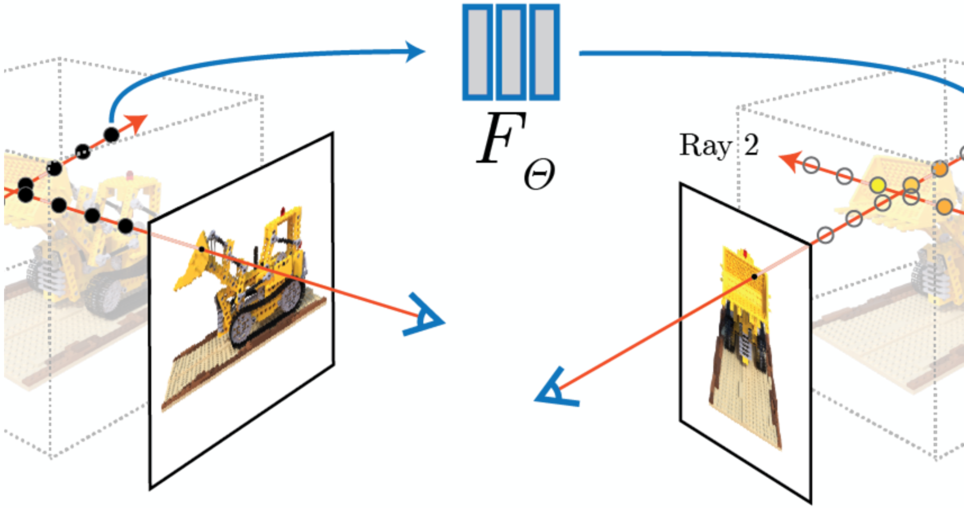
John von Neumann



Inspired from NeRF:
(neural radiance fields)

The weights of a neural network are able to store highly detailed information on complex 3D scene

Mildenhall, Ben, et al. "Nerf: Representing scenes as neural radiance fields for view synthesis." *European conference on computer vision*. Springer, Cham, 2020.

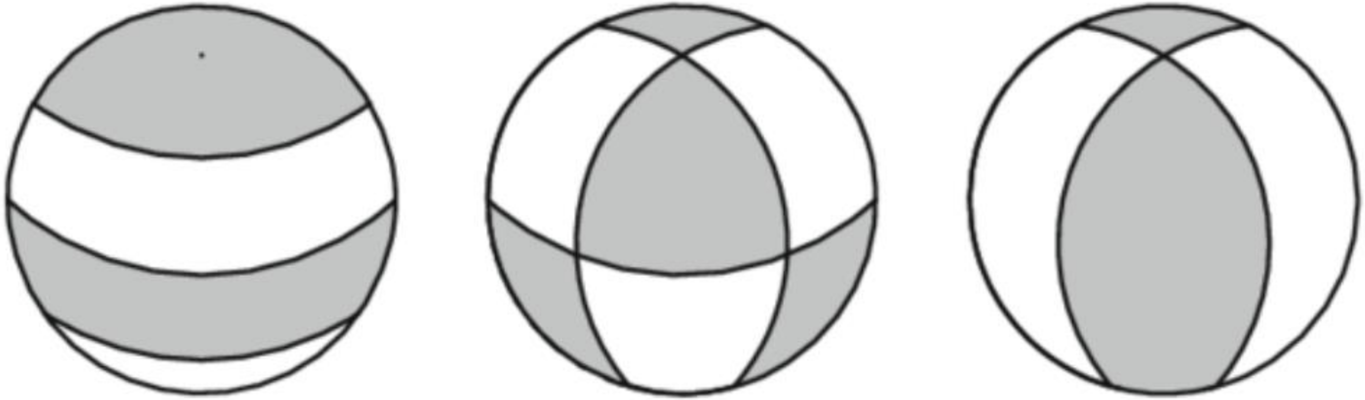


geodesyNETs

Izzo, Dario, and Pablo Gómez. "Geodesy of irregular small bodies via neural density fields: geodesyNets." *arXiv preprint arXiv:2105.13031* (2021).

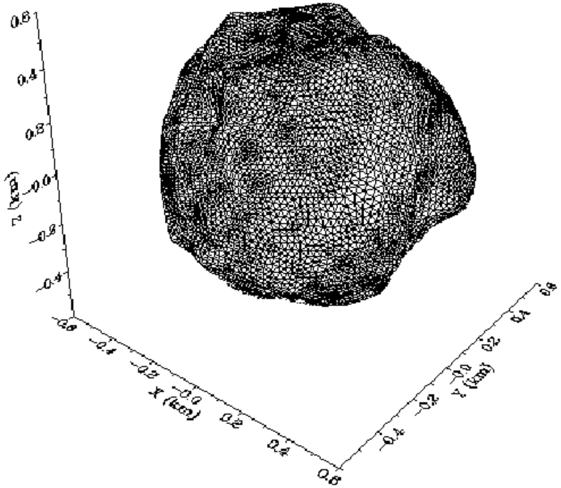
von Looz, Moritz, Pablo Gomez, and Dario Izzo. "Study of the asteroid Bennu using geodesyANNs and Osiris-Rex data." *arXiv preprint arXiv:2109.14427* (2021).

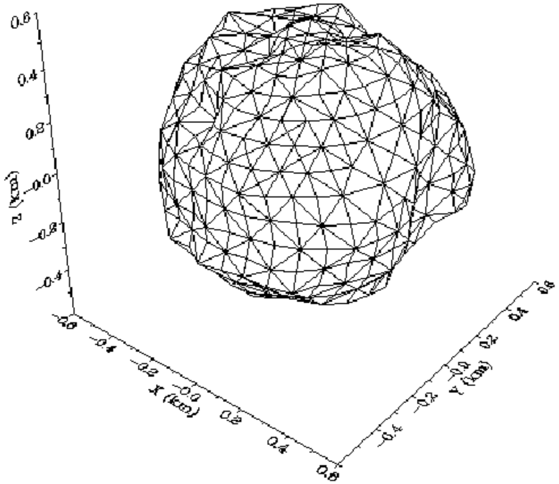
Spherical harmonics - (⅔)



(a) n=3 (b) n=3, m=2 (c) n=m=2

Polyhedral gravity (2/2)

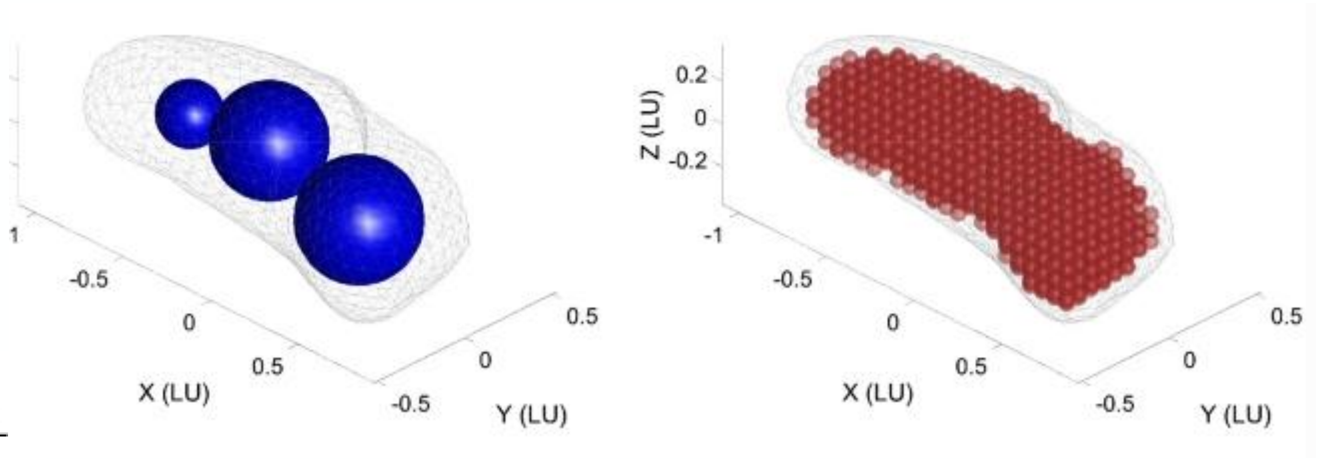




code: https://github.com/esa/polyhedral-gravity-model

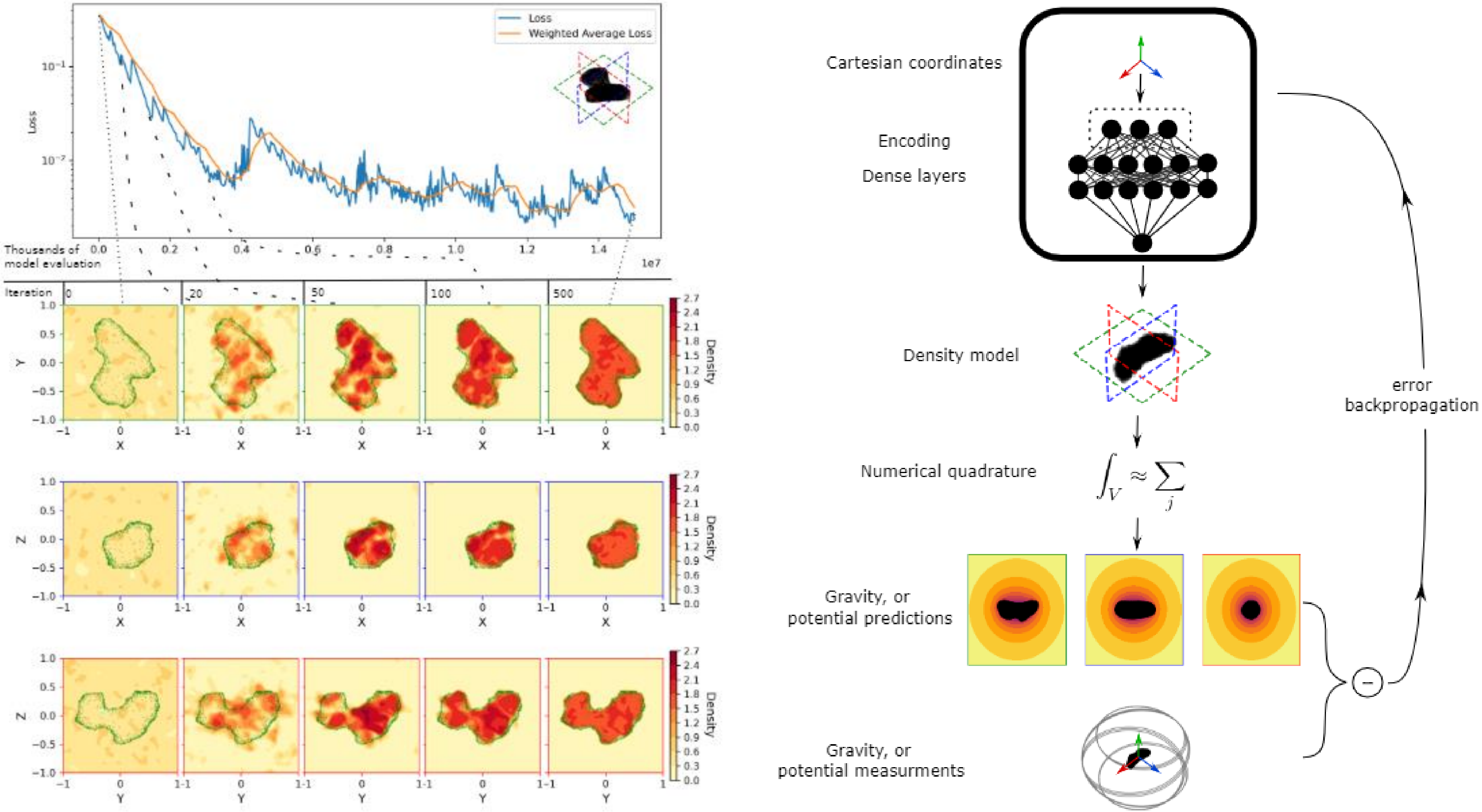
Relies and needs on the asteroid shape, unable to see inside.

3.Mascon models



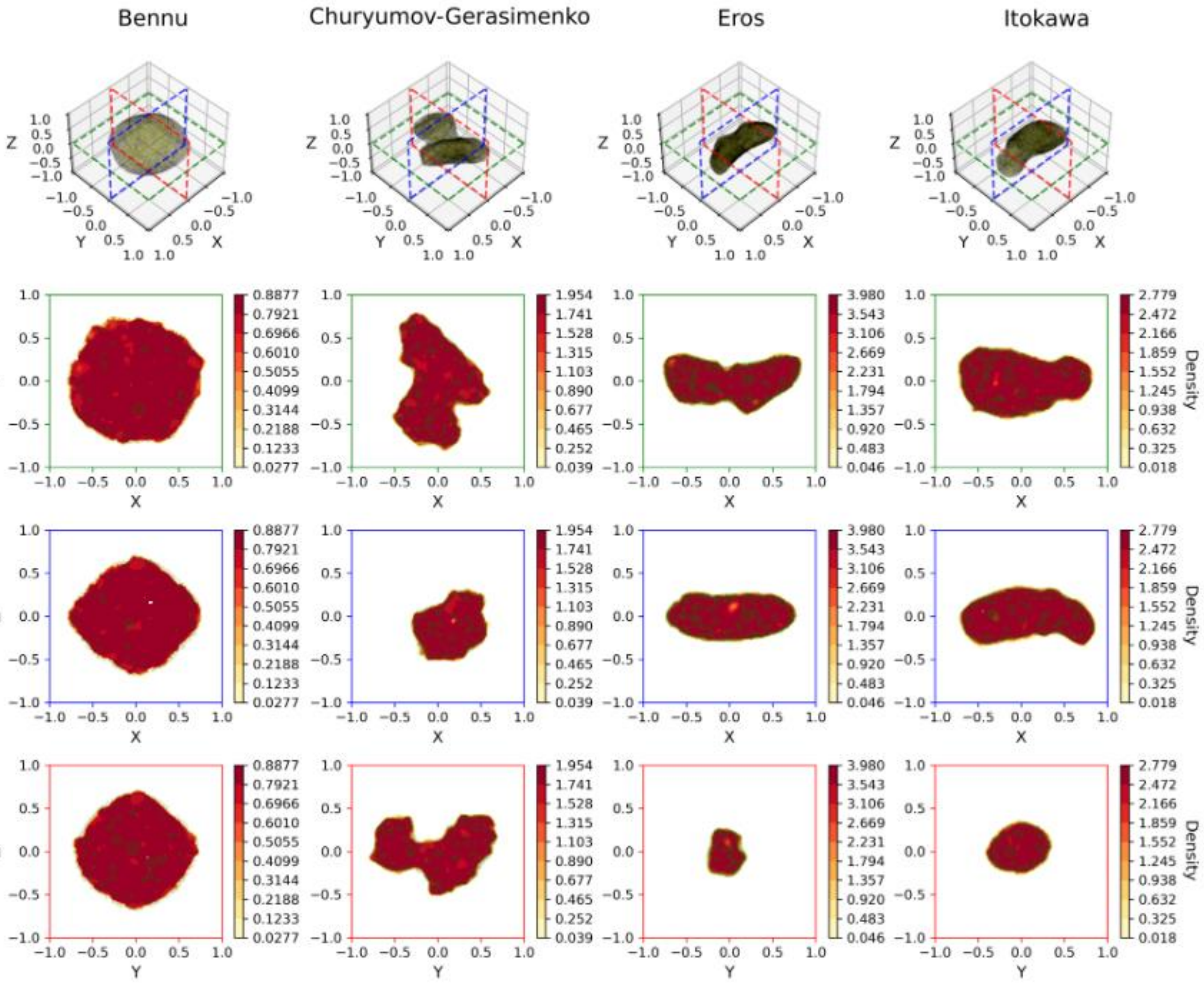
$$U(r) = \sum_{i=0}^{N} \frac{m_i}{r_i}$$

Great flexibility but poor precision next to the surface and needs shape information.


Loss
Weighted Average Loss
Thousands of model evaluation
1e7
Iteration
0
20
50
100
500
Density
Cartesian coordinates
Encoding
Dense layers
Density model
Numerical quadrature
$\int_V \approx \sum_j$
Gravity, or potential predictions
Gravity, or potential measurments
error backpropagation

	Approach			
	Masc.	Harm.	Poly.	geodesyNets
Differentiable	✗	✓	✓	✓
Inside Brillouin sphere	✓	✗	✓	✓
Heterogeneous densities	✓	✓	✗	✓
Shape model not needed	✓	✓	✗	✓
Can utilize shape model	✓	✗	✓	✓
Accurate in the near field	✗	✓	✓	✓

✓


Bennu
Churyumov-Gerasimenko
Eros
Itokawa
X
Y
Z
Density
0.8877
0.7921
0.6966
0.6010
0.5055
0.4099
0.3144
0.2188
0.1233
0.0277
1.954
1.741
1.528
1.315
1.103
0.890
0.677
0.465
0.252
0.039
3.980
3.543
3.106
2.669
2.231
1.794
1.357
0.920
0.483
0.046
2.779
2.472
2.166
1.859
1.552
1.245
0.938
0.632
0.325
0.018

Visualizing the Neural Density field

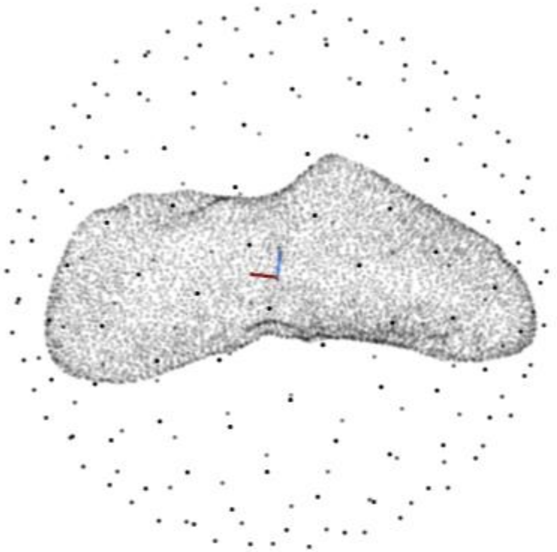
Torus

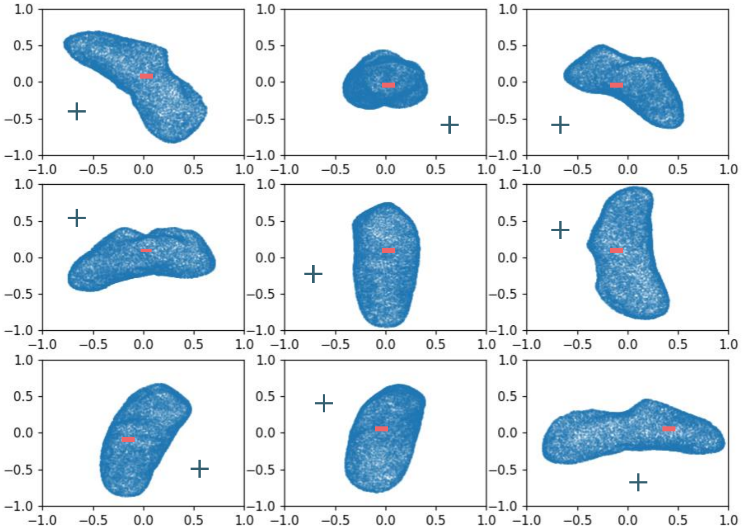
Eclipse Nets

… also an implicit neural representation

Biscani, Francesco, and Dario Izzo. "Reliable event detection for Taylor methods in astrodynamics." *Monthly Notices of the Royal Astronomical Society* 513.4 (2022): 4833-4844.

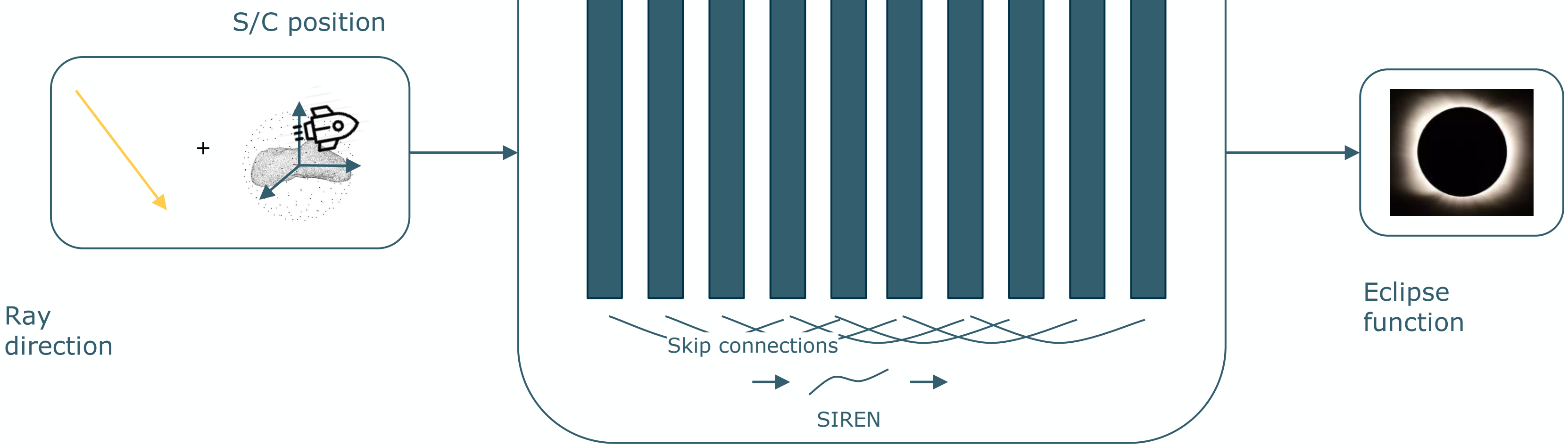
The eclipse function: $F(\mathbf{r}, \hat{\mathbf{i}}_S)$

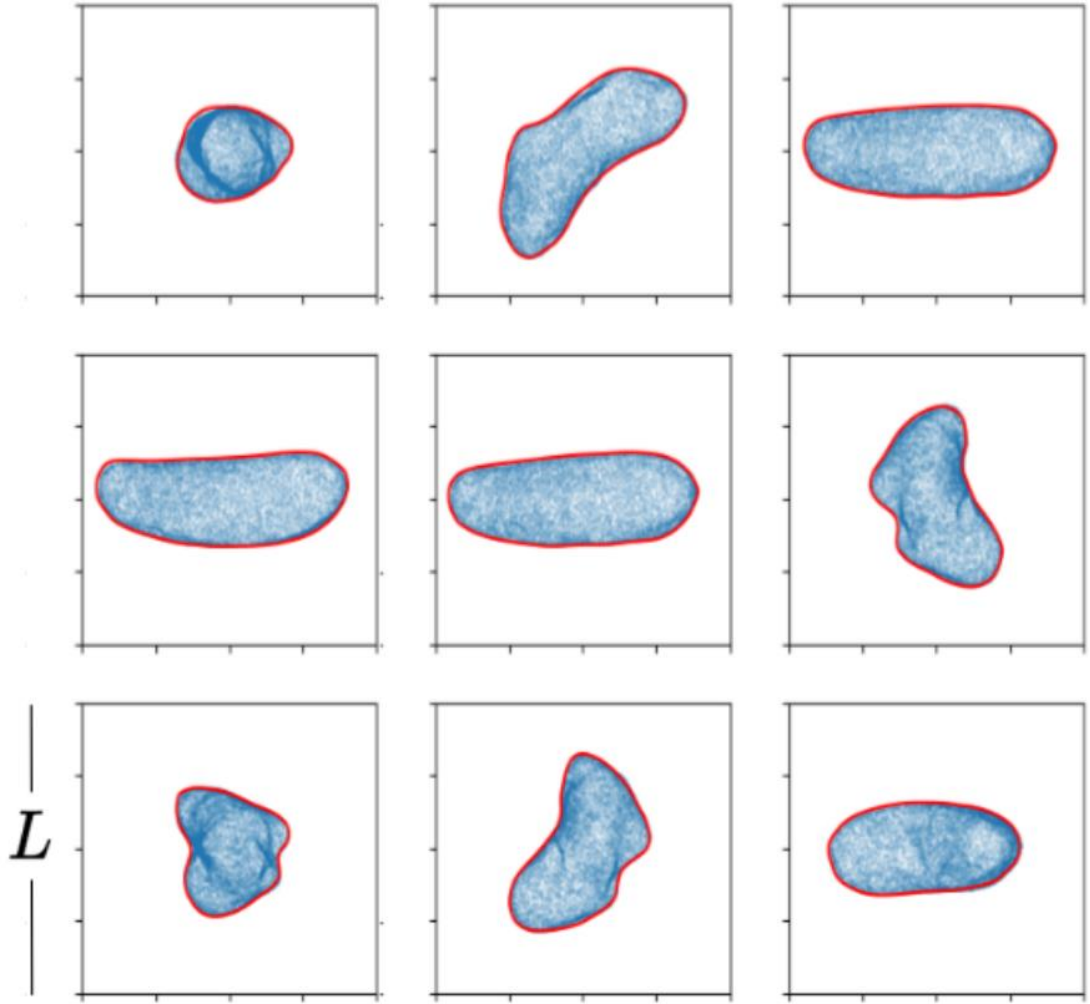




The eclipse function can be used to determine the presence or absence of solar radiation pressure.

eclipseNet


S/C position
+
Ray
direction
dense layers
Skip connections
SIREN
Eclipse
function


L

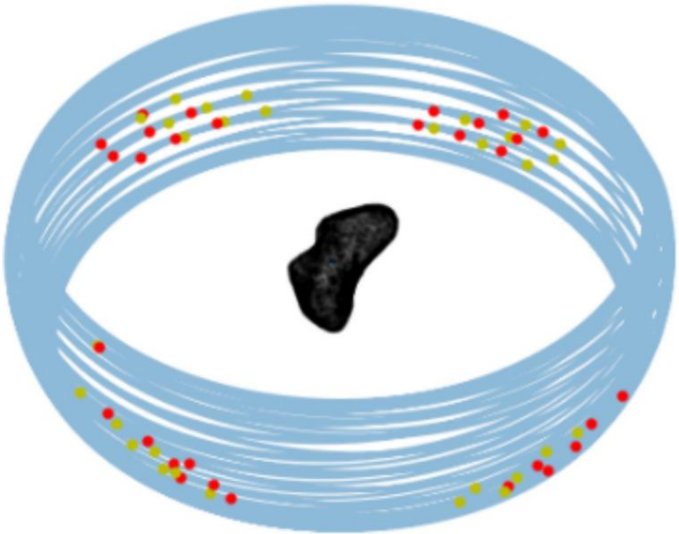
$$\ddot{\mathbf{r}} = -G\sum_{j=0}^{N}\frac{m_j}{|\mathbf{r}-\mathbf{r}_j|^3}(\mathbf{r}-\mathbf{r}_j) - 2\omega\times\mathbf{v} - \omega\times\omega\times\mathbf{r} - \eta\nu(\mathbf{r})\hat{\mathbf{i}}_S(t),$$

$$\nu(\mathbf{r}) = 1$$

no penubra

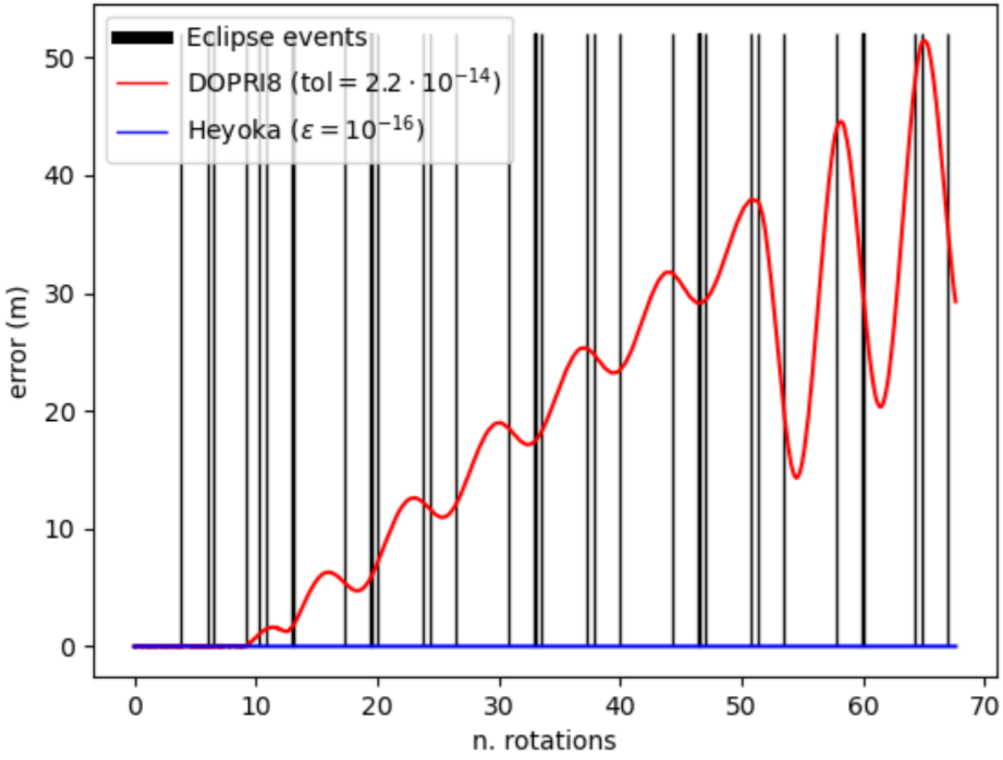
$$\eta(\mathbf{r}, \mathbf{i}_S) = H(F(\mathbf{r}, \mathbf{i}_S))$$

Heaviside function



We obtain a "neural" ODE on top of which to perform event detection -> heyoka!

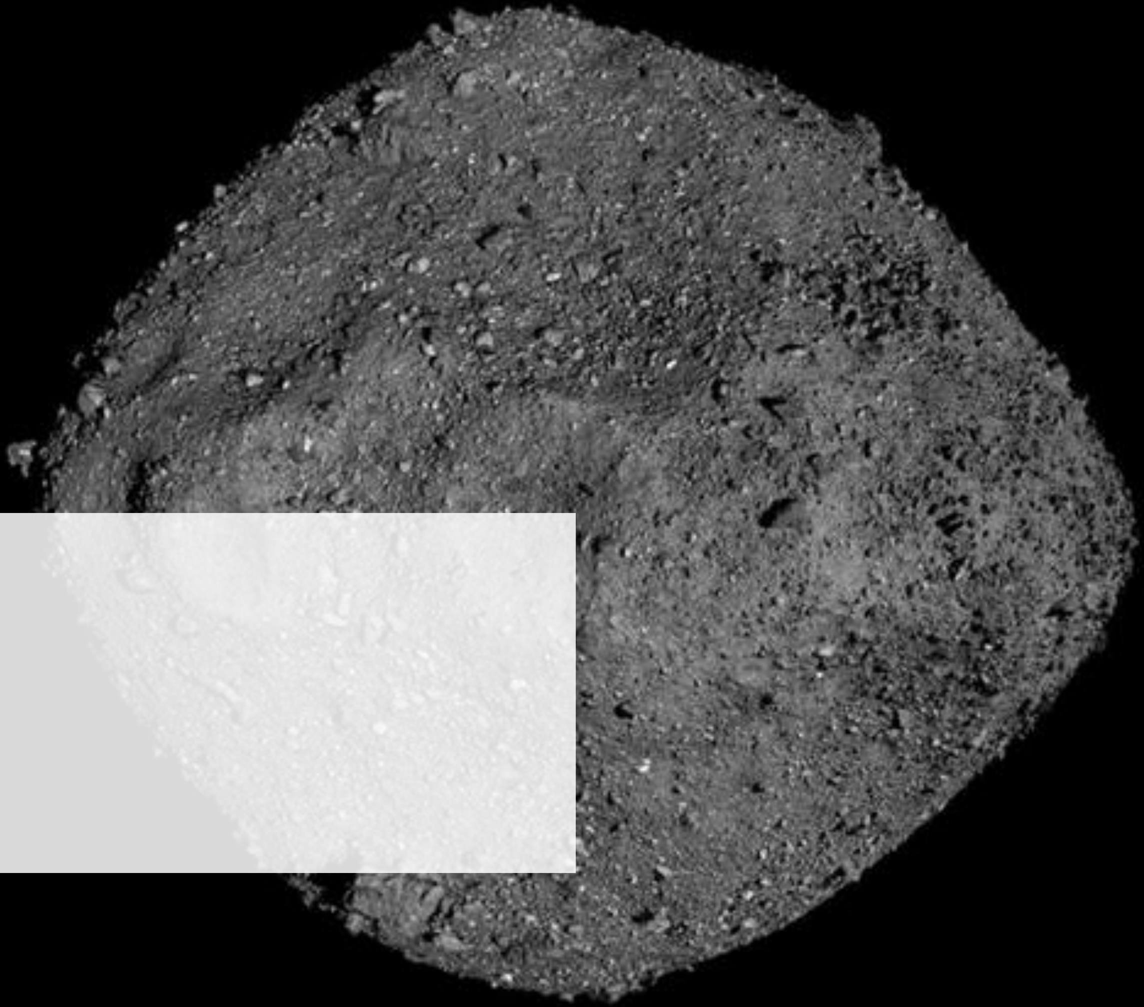
Biscani, Francesco, and Dario Izzo. "Reliable event detection for Taylor methods in astrodynamics." *Monthly Notices of the Royal Astronomical Society* 513.4 (2022): 4833-4844.

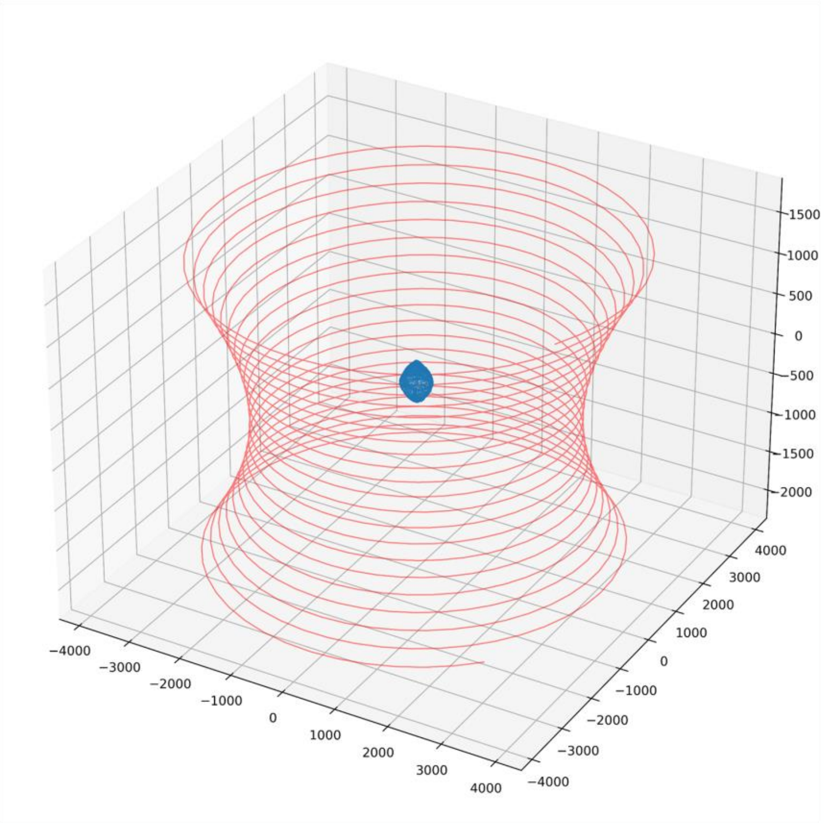


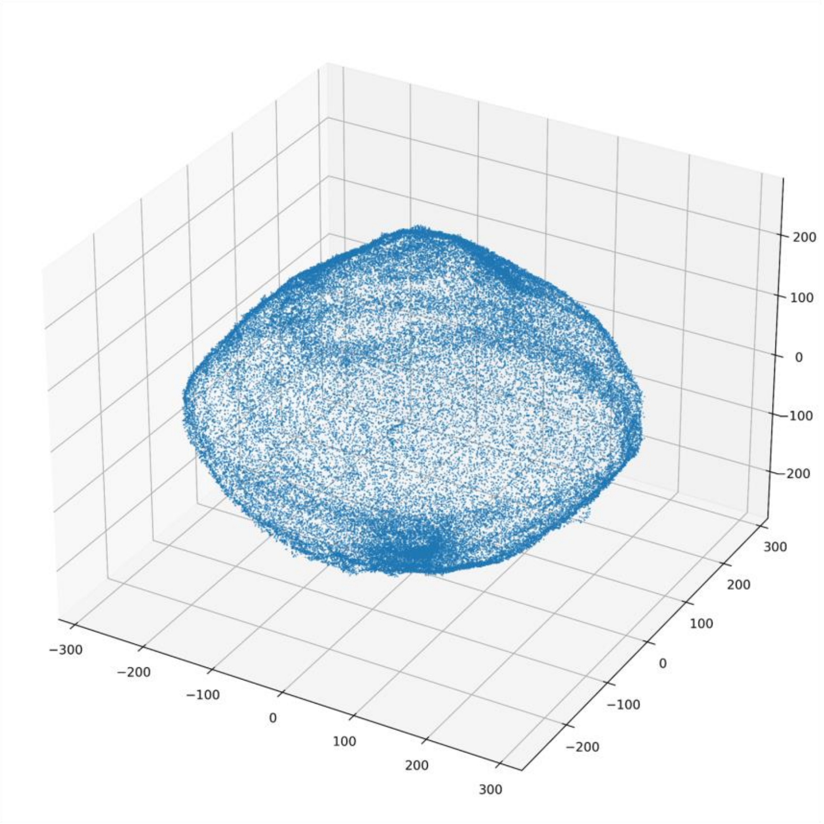
Eclipse events
DOPRI8 (tol = 2.2 · 10^−14)
Heyoka (ε = 10^−16)
error (m)
0
10
20
30
40
50
0
10
20
30
40
50
60
70
n. rotations

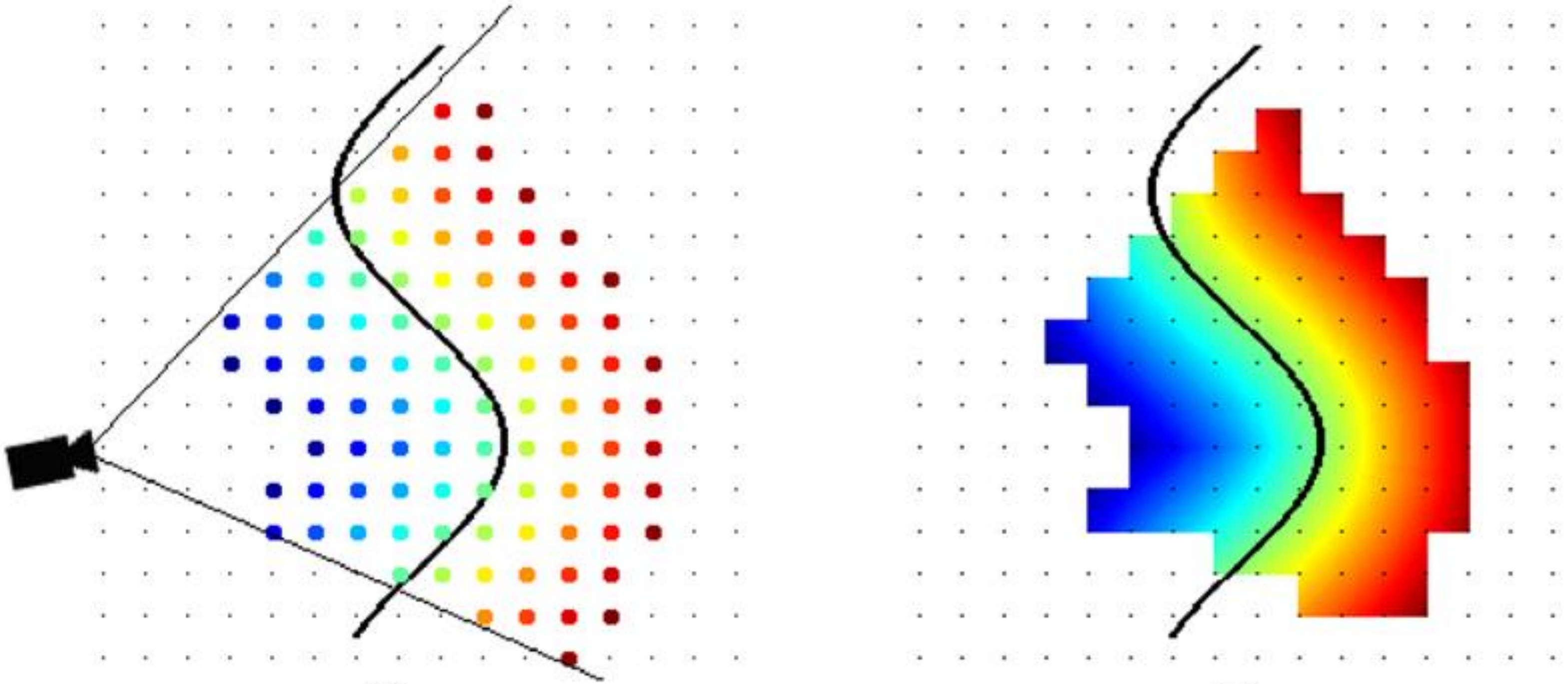
Optimization on Real Mission Data

Bennu

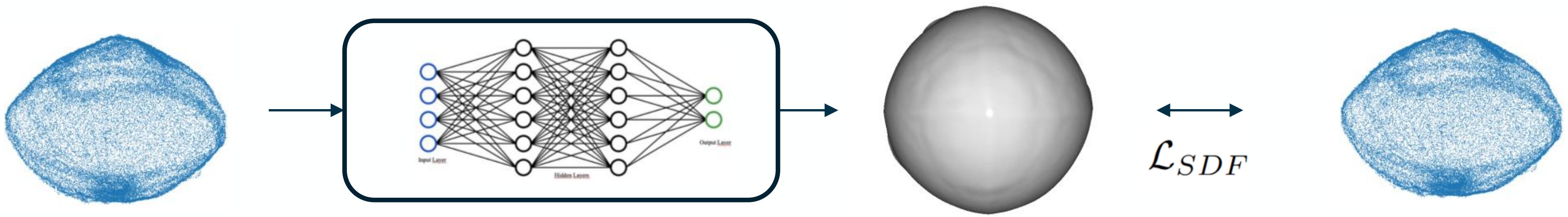



1500
1000
500
0
−500
−1000
−1500
−2000
−4000
−3000
−2000
−1000
0
1000
2000
3000
4000
4000
3000
2000
1000
0
−1000
−2000
−3000
−4000

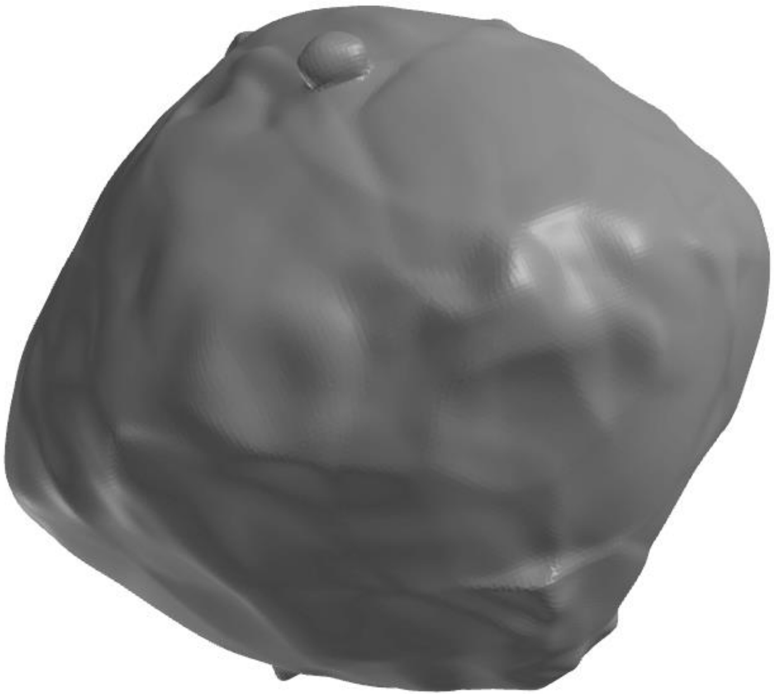

200
100
0
−100
−200
−300
−200
−100
0
100
200
300
300
200
100
0
−100
−200

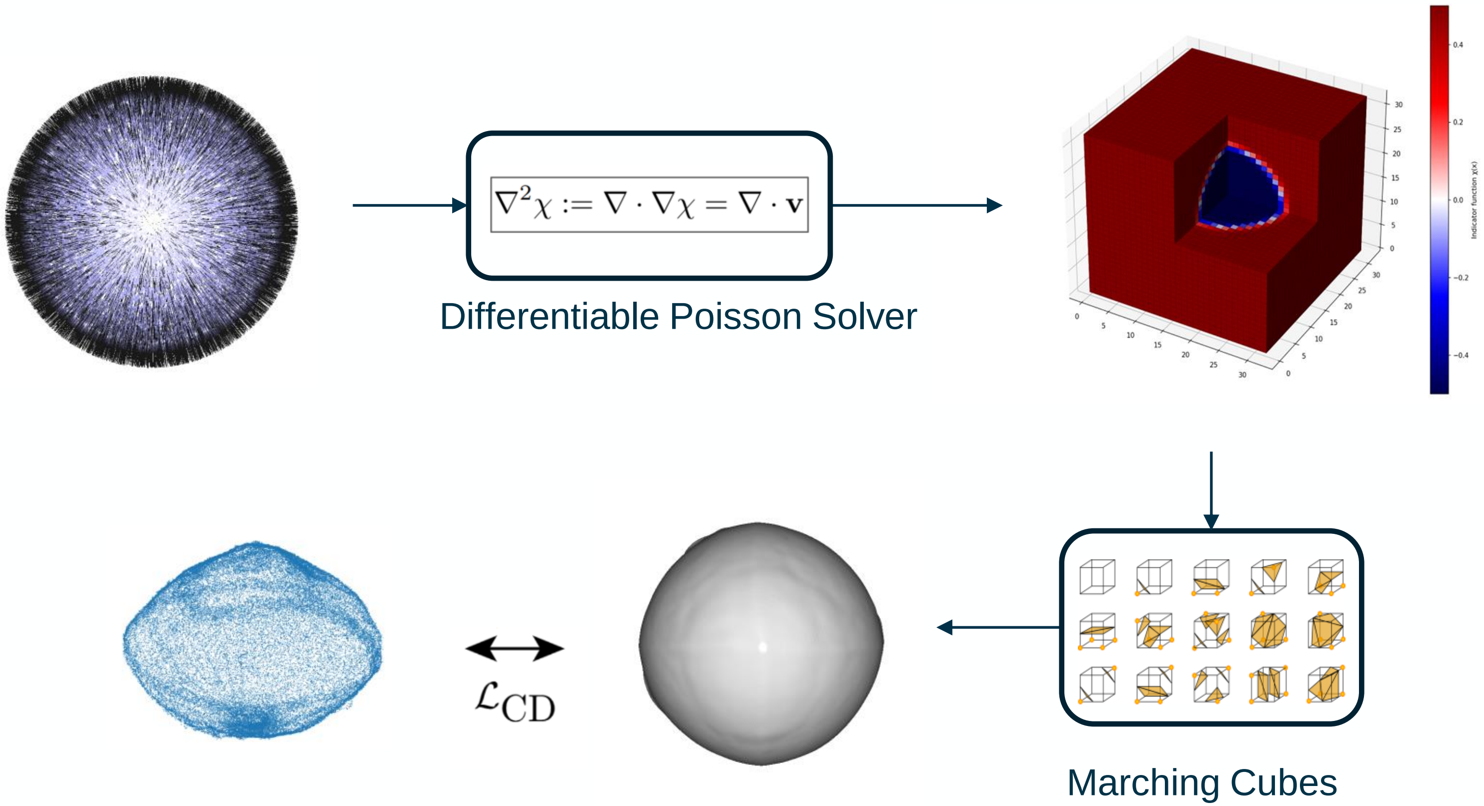


(a) (b)

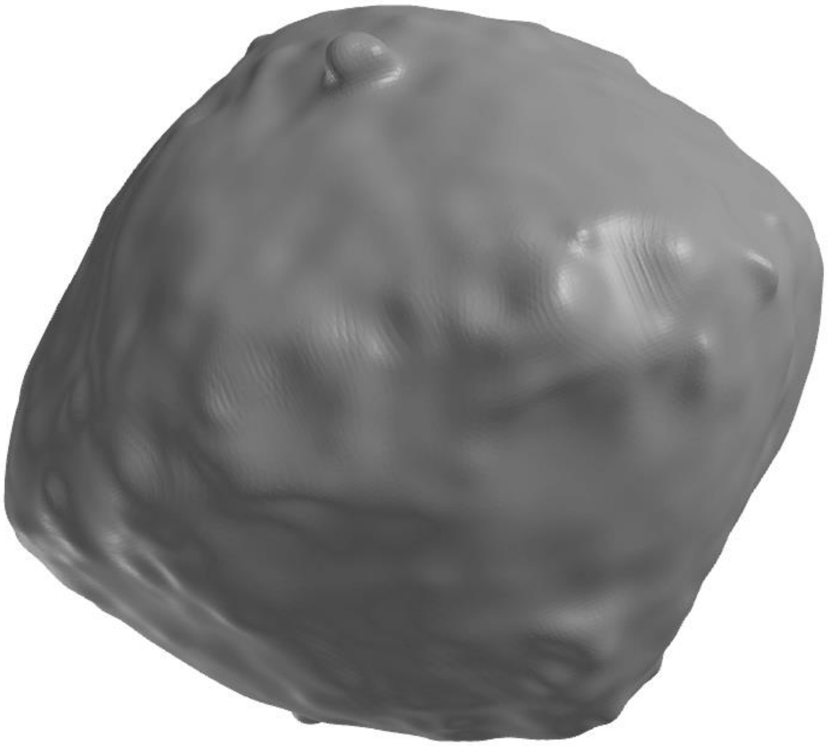


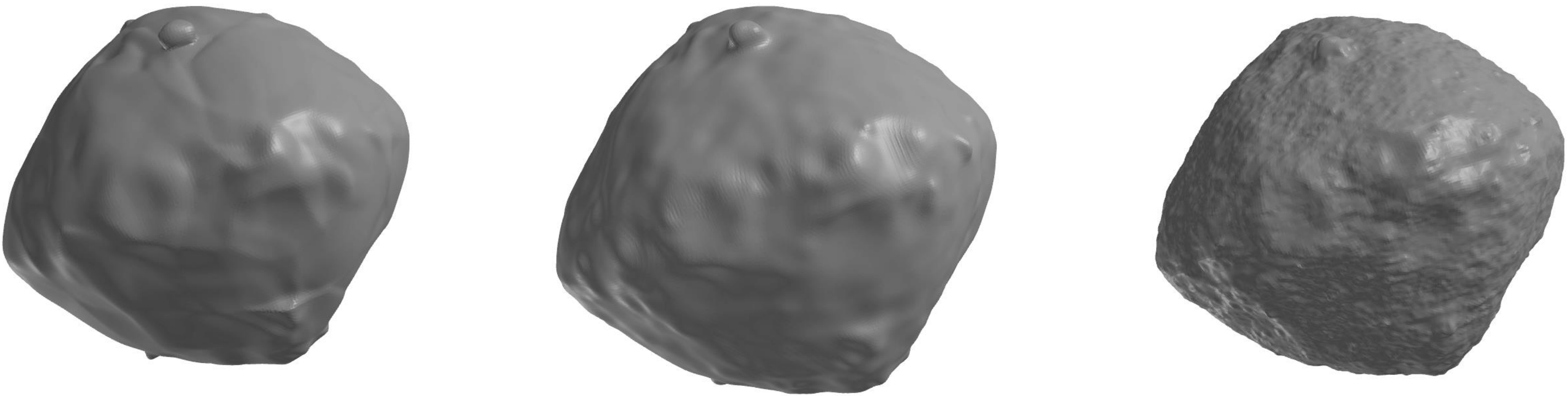
Implicit Neural Representation

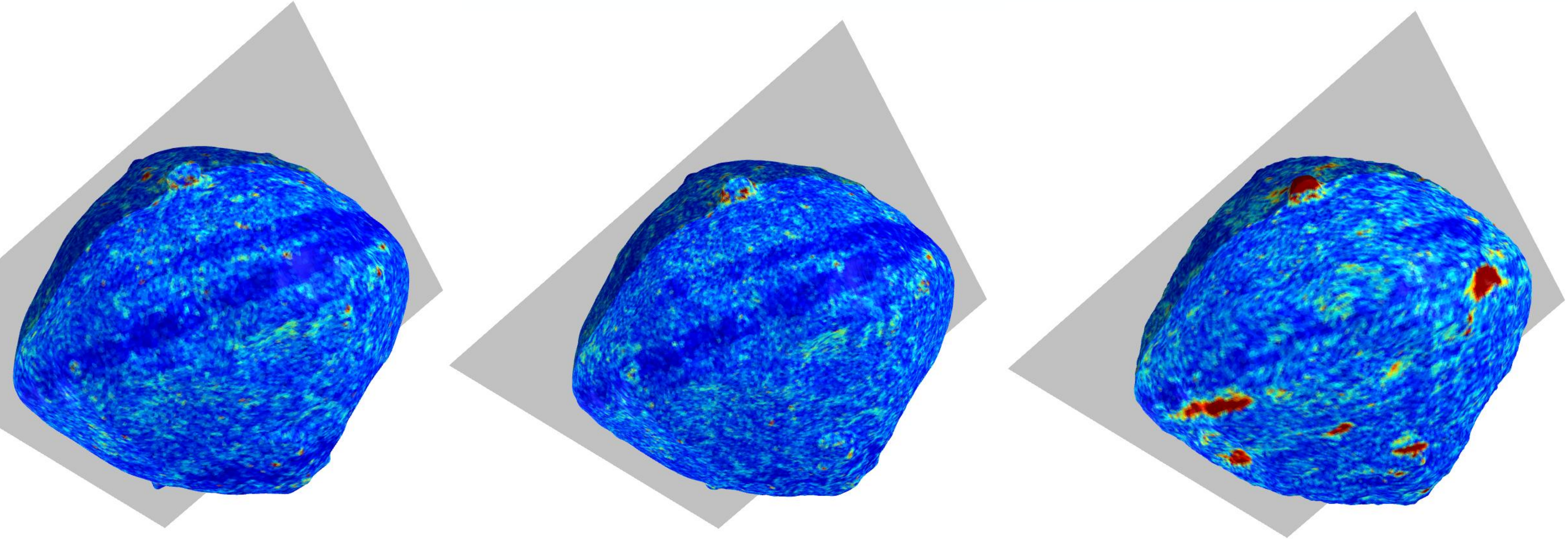




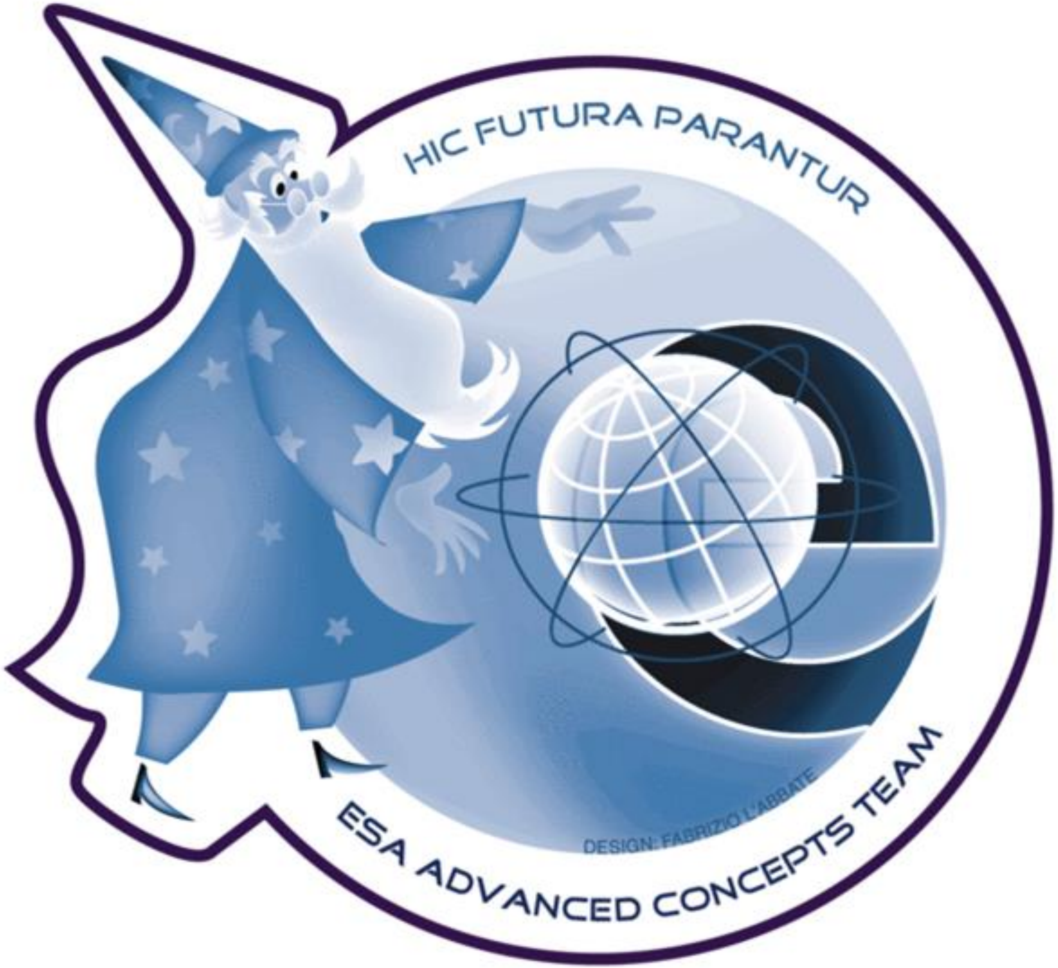
$\nabla^2\chi := \nabla \cdot \nabla\chi = \nabla \cdot \mathbf{v}$
Differentiable Poisson Solver
Indicator function χ(x)
$\mathcal{L}_{\mathrm{CD}}$
Marching Cubes

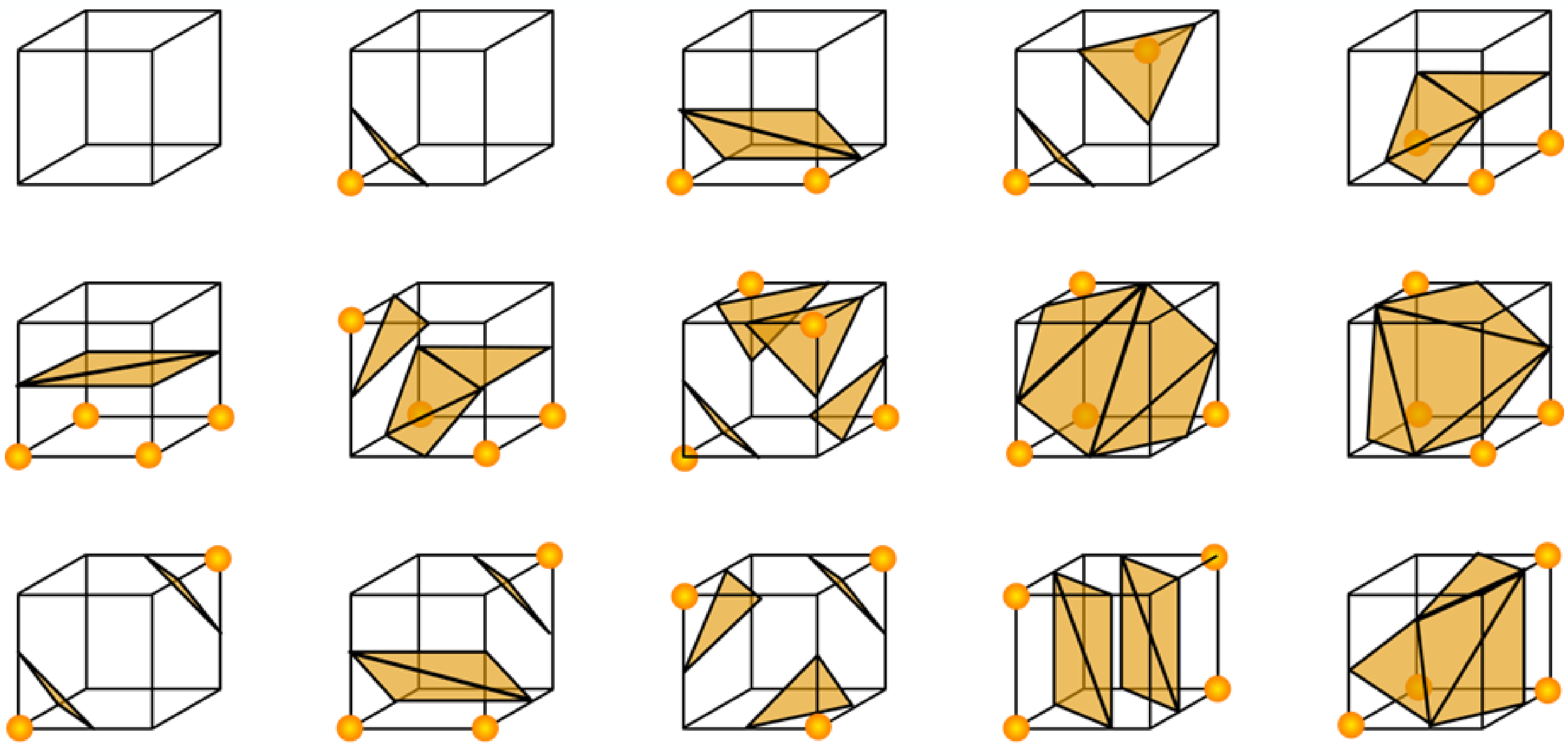






Thank you for listening!



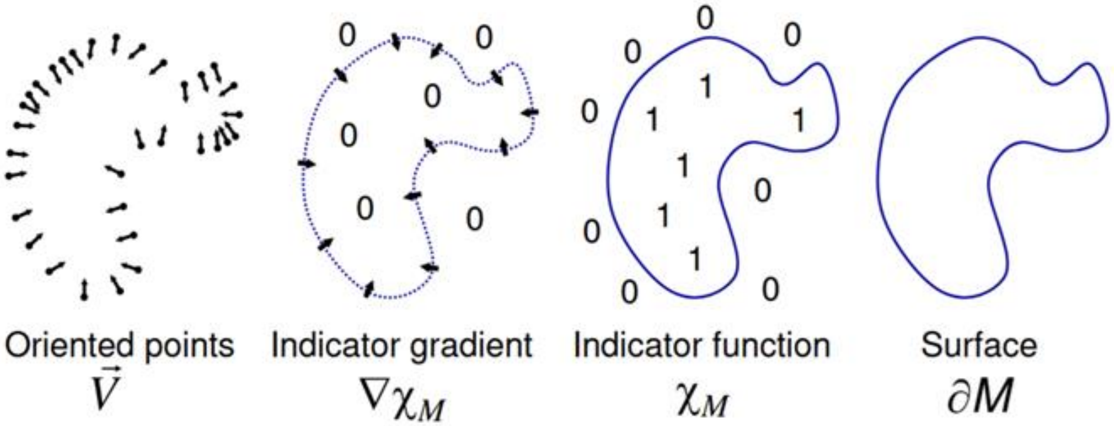


A signed distance function (SDF) determines the distance of a given point, x, from the boundary of Ω

$$f(x) = \begin{cases} d(x, \partial\Omega) & if\ x \in \Omega \\ -d(x, \partial\Omega) & if\ x \in \Omega^c \end{cases}$$

The zero-level set (i.e $f(x)=0$) represents the surface of the domain – this is a form of implicit representation.

An interesting point is that the gradient of the SDF is one everywhere, i.e: take one step away and SDF increases by one.



$$\mathbf{v}(\mathbf{x}) = \sum_{(\mathbf{c}_i, \mathbf{n}_i) \in \{\mathbf{p}\}} \delta(\mathbf{x} - \mathbf{c}_i, \mathbf{n}_i), \text{ where } \delta(\mathbf{x}, \mathbf{n}) = \{\mathbf{n} \text{ if } \mathbf{x} = 0 \text{ and } 0 \text{ otherwise}\}$$

indicator function $\chi(\mathbf{x})$

$$\nabla^2 \chi := \nabla \cdot \nabla \chi = \nabla \cdot \mathbf{v}$$

$$\chi = (\nabla^2)^{-1} \nabla \cdot \mathbf{v} \quad \text{s.t.} \quad \chi|_{\mathbf{x} \in \{\mathbf{c}\}} = 0 \quad \text{and} \quad \text{abs}(\chi|_{\mathbf{x}=0}) = m$$

$$\mathrm{FFT}(\nabla \cdot \mathbf{v}) = 2\pi i(\mathbf{u} \cdot \tilde{\mathbf{v}})$$

$$\mathrm{FFT}(\nabla^2) = -4\pi^2||\mathbf{u}||^2$$

$$\tilde{\chi} = \tilde{g}_{\sigma,r}(\mathbf{u})\frac{i\mathbf{u} \odot \tilde{\mathbf{v}}}{-2\pi||\mathbf{u}||^2} \qquad \tilde{g}_{\sigma,r}(\mathbf{u}) = \exp\left(-2\frac{\sigma^2||\mathbf{u}||^2}{r^2}\right)$$

$$\chi' = \mathrm{IFFT}(\tilde{\chi})$$