

The puzzling dynamic evolution of defunct satellites: a challenge for Active Debris Removal missions

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End-of-Life Management & Zero Debris
Design for Removal
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Included but not to be presented:

Annex A. PMD simulators examples

Annex B. PMD simulations examples

Annex C. HW level verification of a PMD system (short-circuited Magnetic Torquers)

Annex D. TOPEX Poseidon tentative analysis

1. Introduction

- **A big challenge driving the complexity of the rendezvous and capture of debris is its tumbling motion**
 - Observations of non-operational LEO satellites often show angular rates above 2 deg/s
 - The prediction and estimation of the angular rates of defunct satellites to be captured is therefore crucial for the design of the chaser and to confirm the feasibility of these critical operations

- **This paper shares what we have been learning over the past years about the dynamic evolution of defunct satellites in Low Earth Orbit, how complex it is and how to do this assessment properly**
 - Can we explain the challenging diversity of defunct satellites rotational motion in LEO? (slide 4)
 - Trying to understand the dynamics of the spinning phase (slides 5 to 7 and Annex D)
 - How efficiently can energy dissipation devices damp the angular rates before an ADR mission?
 - Sun Radiation Pressure spin-averaged torques and mitigation action (slide 8)
 - Spin rate evolution analytical prediction (slides 9, 10)
 - Which verification process to assess the long-term dynamic evolution?
 - Preliminary guidelines prepared for ESA SDM Handbook on the specific subject of Magnetic Detumbling performance validation and verification (slides 11 to 15 and Annex A and B)
 - Conclusions, acknowledgements and references (slides 16,17)
 - Some application examples in Annexes A, B, C and D (slides 18 to 27)

2. The puzzling dynamic evolution of defunct satellites

□ The challenging diversity of defunct satellites rotational motion in LEO

- A hypothetical ideal situation for ADR: in case of unsuccessful deorbiting, the S/C would reach autonomously a stable Earth-locked attitude thanks to the Gravity Gradient torques and energy dissipation (like the Moon showing the same visible face towards the Earth).
- Unfortunately, attitude reconstructions of defunct satellites reveal a variety of complex evolutions, ending up sometimes with significant angular rates, even after a successful decommissioning.

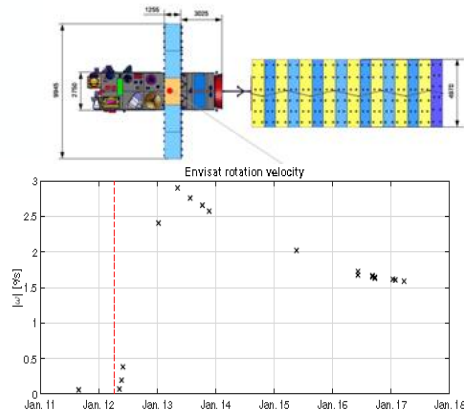
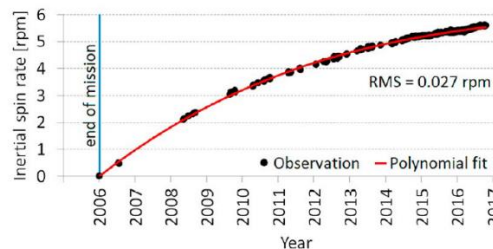
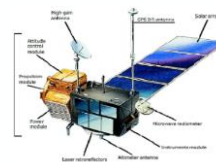
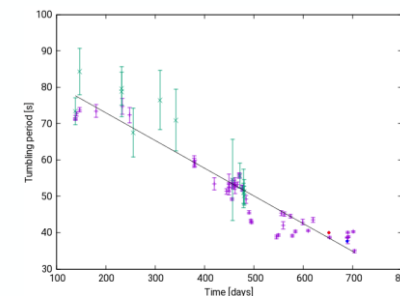


Figure 4. Angular velocity derived by TIRA measurements from August 2011 to March 2017. The red dashed line indicates the date of loss of contact, 8 April 2012.

Envisat : up to 3%/s and down to 1.5%/s (TBC)



TOPEX/Poseidon: up to 36%/s (TBC)



**Jason 2 spin periods
i.e. from 4%/s to 10%/s over 2 years**

□ References:

- Kucharski, D. et al., Attitude and Spin Period of Space Debris Envisat Measured by Satellite Laser Ranging. IEEE Transactions on Geoscience and Remote Sensing
- Sommer, S. et al., Temporal analysis of Envisat's rotational motion, Proc. 7th European Conference on Space Debris, Darmstadt, Germany, 18–21 April 2017
- Kucharski, D. et al: Photon Pressure Force on Space Debris TOPEX/Poseidon Measured by Satellite Laser Ranging - Earth and Space Science, 4, 661–668.
- Vananti, A. et al: Multi-sensor Space Object Tracking for Tumbling Motion Characterization - Proc. 2nd NEO and Debris Detection Conference, Darmstadt, 24-26 Jan 2023

3. Trying to understand the spinning phase (1/3)

Consequences in view of Active Debris Removal missions:

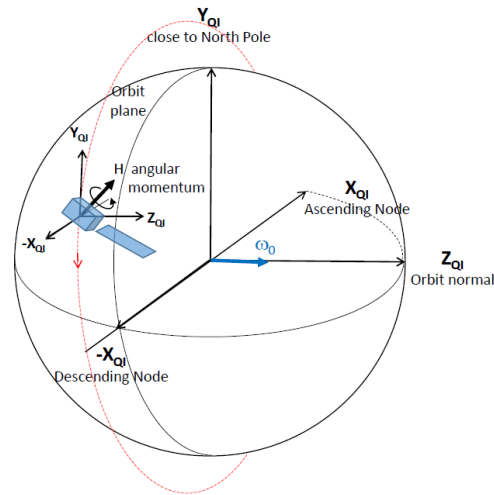
- Compatibility of ADR design for RV and Capture with a tumbling or spinning non-cooperative defunct satellite
- Necessity to master long-term dynamics evolution of defunct satellites, particularly the spinning configuration

Long-term evolution of the angular momentum $\vec{H} = I_z \vec{\omega}_z$ under 2 conservative torques

- Slow evolution of \vec{H} driven by the spin-averaged and orbit-averaged values of the external torques :

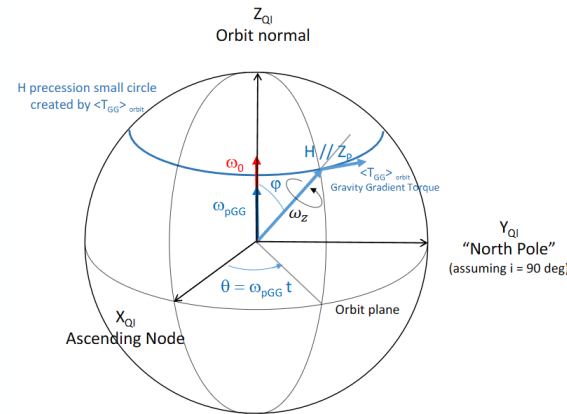
$$\langle \vec{T}_{EXT} \rangle_{spin} = \frac{1}{T_{spin}} \int_0^{T_{spin}} \vec{T}_{EXT}(t) dt \text{ and } \langle \vec{T}_{EXT} \rangle_{orbit} = \frac{1}{T_{orbit}} \int_0^{T_{orbit}} \vec{T}_{EXT}(t) dt \text{ (integrated in inertial reference frame)}$$

- Angular rate ω_z around major principal axis Z_p of inertia I_z and negligible nutation (ω_x and ω_y)
- Quasi polar circular orbit with orbital rate ω_0 , the obliquity φ is the angle between $\vec{H} = I_z \vec{\omega}_z$ and the orbit normal \vec{Z}_{QI}



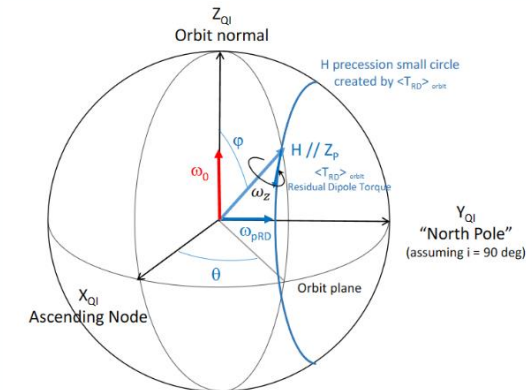
Spinning defunct satellite in (quasi) polar circular Low Earth Orbit

Axes rotation



Impact of Gravity Gradient: Precession around orbit normal constant spin rate, constant obliquity

$$\langle \vec{T}_{GG} \rangle_{orbit} = \frac{3\omega_0^2}{2} \left[I_z - \left(\frac{I_x + I_y}{2} \right) \right] (\vec{Z}_{QI} \cdot \vec{Z}_p) (\vec{Z}_p \times \vec{Z}_{QI})$$



Impact of S/C Residual Magnetic Dipole: Precession around the Earth Pole (up and down) constant spin rate, modulation of obliquity

$$\langle \vec{T}_{RD} \rangle_{orbit} = \frac{\mathcal{M}_{RDz} B_{eq}}{2} (\vec{Y}_{QI} \times \vec{Z}_p)$$

3. Trying to understand the spinning phase (2/3)

Passive Magnetic Detumbling (PMD)

- To damp angular rates, kinetic energy dissipation means are necessary but not always sufficient
 - Fluid dampers cannot modify the angular momentum! => flat spin transition towards major principal axis and nutation damping but any significant spin rate will remain
- External torques created by the interaction between the Earth magnetic field and conductive structures

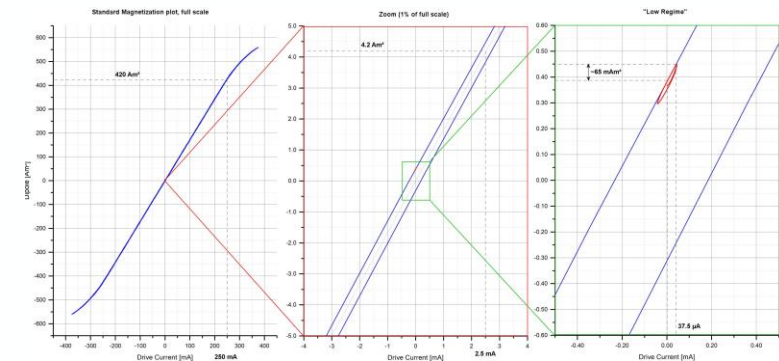
- Torque characterized by a magnetic tensor $\bar{\bar{M}}$ such that

$$\vec{T}_{PMD} = - \left[\bar{\bar{M}} \left(\frac{d\vec{B}_{Earth}}{dt} \right)_{sat} \right] \times \vec{B}_{Earth} = - [M(-\vec{\omega} \times \vec{B})] \times \vec{B} - \left[M \left(\frac{d\vec{B}}{dt} \right)_{inertial} \right] \times \vec{B} \quad (\text{rotational and orbital components})$$

- Damping torque in polar orbit $\langle \vec{T}_{PMD} \rangle_{long\ term} \sim - M \frac{5}{2} B_{eq}^2 (1 - 0.45 \sin^2 \varphi) \vec{\omega}_z$

PMD systems:

- Eddy currents circulating in S/C conductive elements
- Automatic short-circuiting of on-board Magnetic Torquers proposed by ESA to complement Eddy currents. Tests by ZARM Technik have confirmed the validity of the mathematical models for extremely small induced currents and magnetic dipoles.(see Annex C)
- ZARM Magnetic Torquers for Copernicus Expansion missions can be optimised for PMD such that $M \sim 2.0 \cdot 10^4 \Omega^{-1} m^4$ to $2.0 \cdot 10^5 \Omega^{-1} m^4$ (both coils short-circuited)
- Dedicated PMD Systems developed by industry (not necessarily characterised by a magnetic tensor)

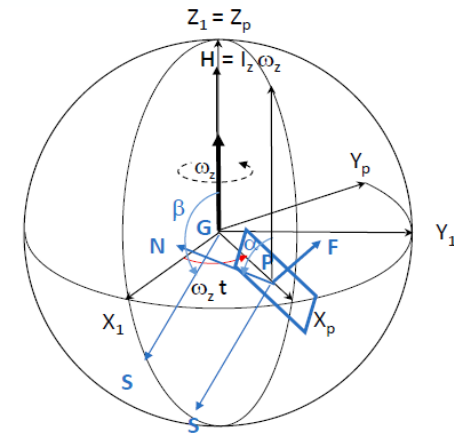


2-step zooming of Magnetic Moment M versus Current i : the magnetic core is correctly excited at low regime as shown by the slope of the small hysteresis loop
 Credit ZARM Technik

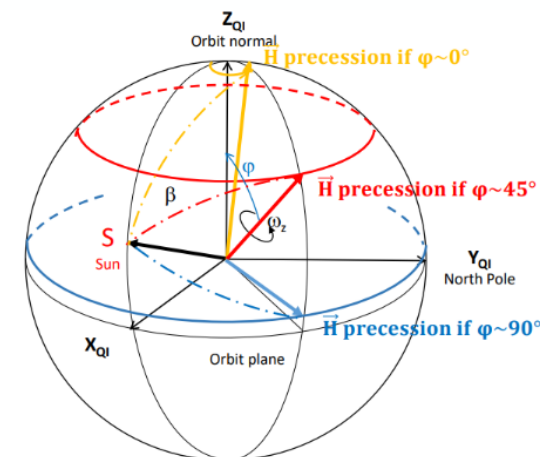
3. Trying to understand the spinning phase (3/3)

Effect on a spinner of the Solar Radiation Pressure (SRP)

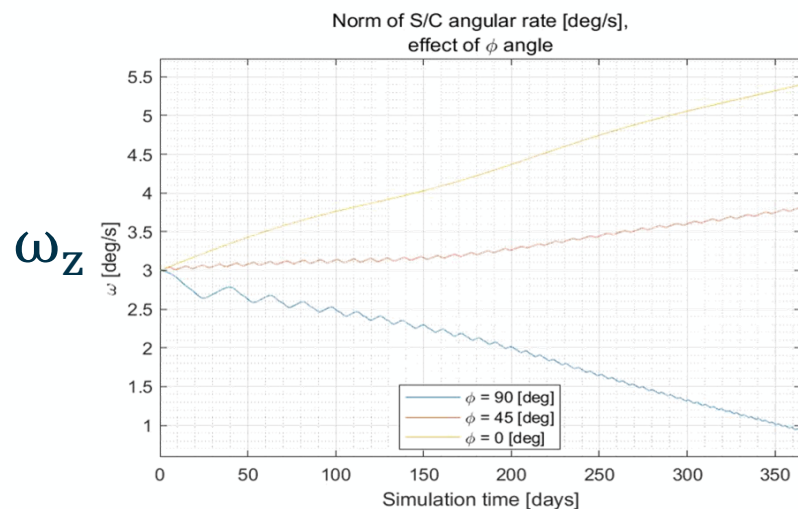
- SRP can spin up or “spin down” space debris exposing imperfect symmetry of revolution to the sun
 - YORP effect named after Yarkovsky-O’Keefe-Radzievskii-Paddack (asteroids spin-up)
 - Tiny but repetitive accumulation of angular momentum created by the spin-averaged $\langle T_{SRP} \rangle_{spin}$
- Sign inversion of $\langle T_{SRP} \rangle_{spin}$ when the sun elevation β crosses the spinner equator
 - The “winner” will depend on the evolution of β , mainly driven by the Gravity Gradient precession, the ratio of illumination above/under the spinner equator $\beta = 90^\circ$



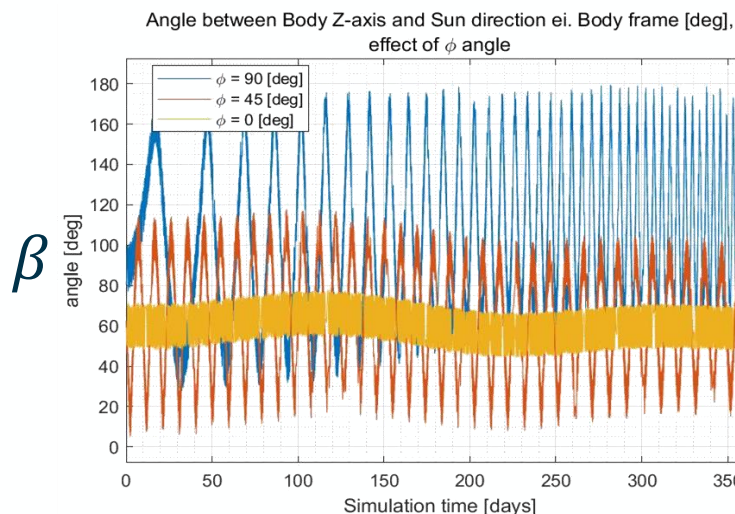
Single lateral Solar Array geometry



Understanding of sun elevation evolution versus obliquity in presence of GG precession



Long term evolution of spin rate related to sun elevation history crossing the spinner equator



4. Efficiency of Passive Magnetic Detumbling (1/3)

❑ PMD can be severely damaged by the Sun Radiation Pressure in case of YORP effect and low authority PMD system : SRP mitigation actions are therefore recommended

❑ Analytical models of $\langle T_{SRP Z} \rangle_{spin}$ for one lateral Solar Array (neglecting central body)

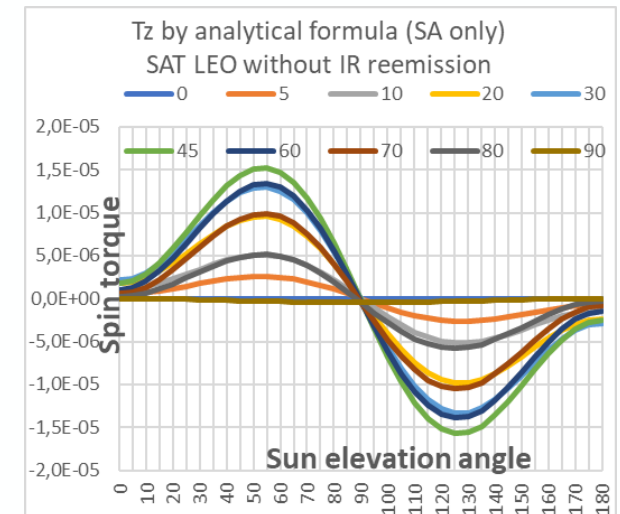
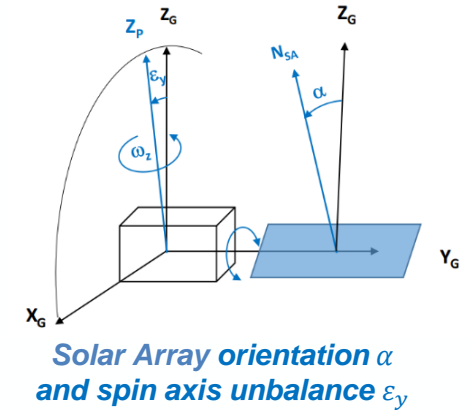
■ Driving parameter: misalignment between the SA Normal and the principal axis Z_p

- $\langle T_{SRP Z} \rangle_{orbit} = \frac{T_{day} \Phi A}{T_{orbit} c} l \sin(\alpha - \varepsilon_y) function(\beta, (\alpha - \varepsilon_y))$
- Maximum torque if $\alpha - \varepsilon_y = 45^\circ$
- Small torque if $\alpha - \varepsilon_y = 90^\circ$ (due to different optical coefficients of front and back faces)
- Zero spin-averaged torque if $\alpha - \varepsilon_y = 0^\circ$ (Solar Array normal parallel to the spin axis),

❑ Mitigation actions:

- Solar Array normal nearly parallel to the major axis of inertia and not at 45°
 - $\langle T_{SRP Z} \rangle_{spin}$ will be divided by a ratio 5 to 10
- Minimization of S/C cross products of inertia such that principal axes misalignment $< 5^\circ$
 - No YORP effect from the Solar Array in case of perfect compensation $\alpha - \varepsilon_y = 0^\circ$
- For a S/C with 2 Solar Arrays, both normal vectors need to be parallel

❑ Simulations are necessary to check if the impact of the central body facets is negligible, and predict the evolution of $\langle T_{SRP Z} \rangle_{orbit}$ and ω_Z with φ, β history

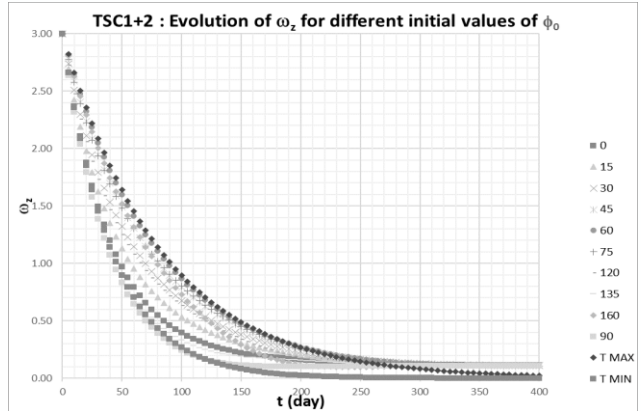


4. Efficiency of Passive Magnetic Detumbling (2/3)

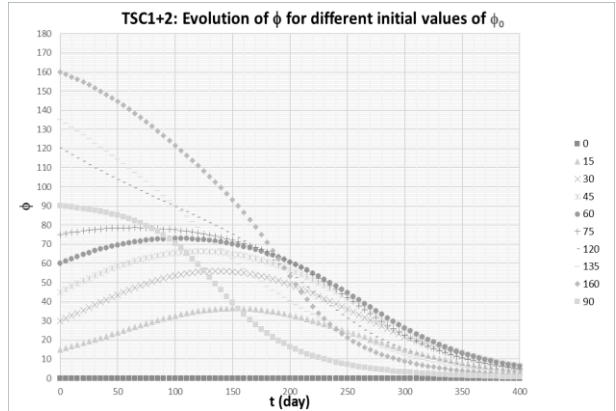
Spin rate evolution analytical prediction

- Typical reqt: “The evolution of the module of the satellite angular rates vector should converge to values lower than 1 deg/s.”
- Case 1: Symmetry of revolution (e.g. rocket or symmetrical S/C) and negligible YORP effect
 - Damping torque always present, varying only in a ratio 1 to 2 with the obliquity φ
 - Tilting torque modifies the obliquity φ , bringing the angular momentum parallel to the orbit normal ($\varphi = 0^\circ$)
 - Spin rate exponential decay till a small value ω_{limit} , with an instantaneous time constant $\tau(\varphi)$ directly related to the Magnetic Tensor M, the spin inertia I_z and the module of the Earth magnetic field at equator B_{eq}

$$\omega_z = (\omega_{z0} - \omega_{limit}) e^{-t/\tau(\varphi)} + \omega_{limit} \text{ with } \tau(\varphi) = \frac{I_z}{M \frac{5}{2} B_{eq}^2 (1 - 0.45 \sin^2 \varphi)} \text{ and } \omega_{limit} = \frac{\frac{9\omega_0}{5} \cos \varphi}{1 - 0.45 \sin^2 \varphi}$$



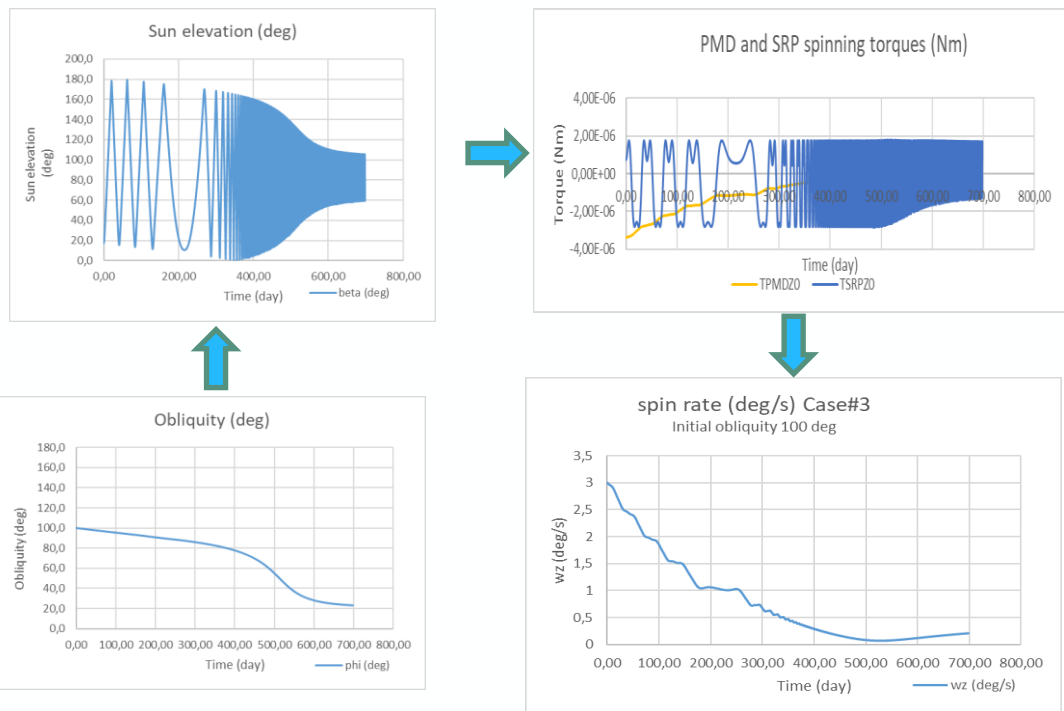
Exponential decay of the spin rate till $\omega_{limit} \sim 2\omega_0 \sim 0.12^\circ/s$



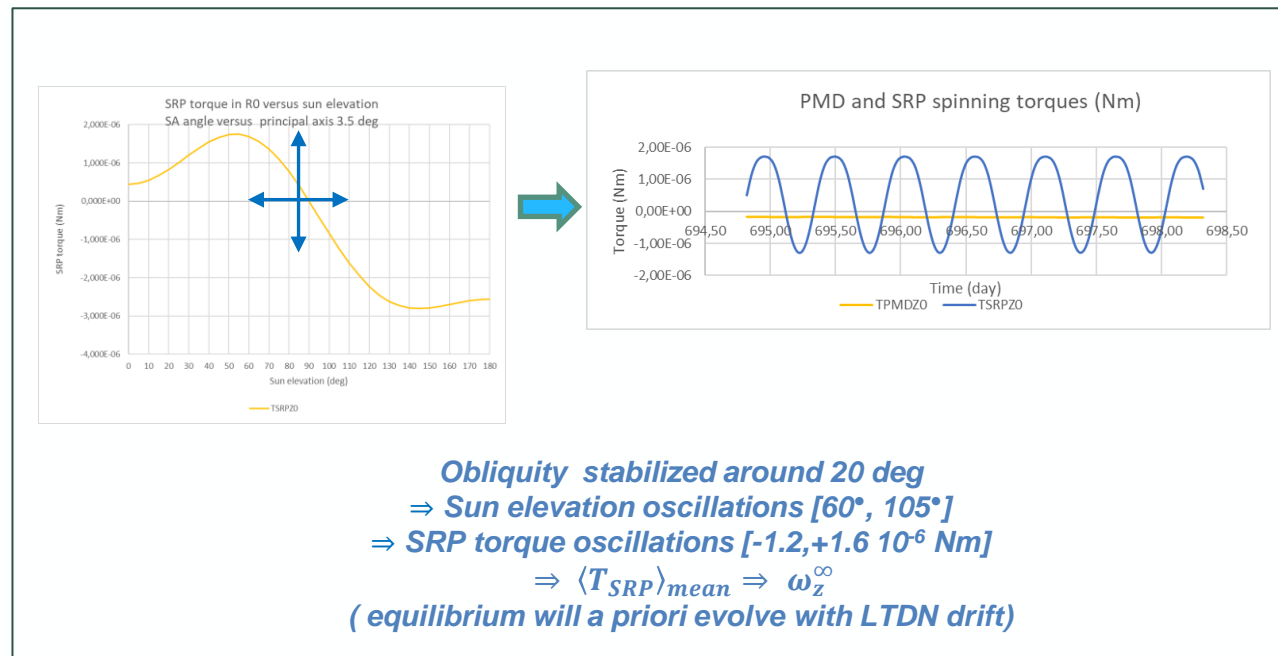
The orbital component of the PMD tilting torque brings the obliquity to 0 deg

4. Efficiency of Passive Magnetic Detumbling (3/3)

- Case 2: No symmetry of revolution wrt principal axes and YORP effect: SRP/PMD torques competition
 - e.g. misalignment of single Solar Array normal wrt principal axis or 2 Solar Arrays not perfectly parallel (wind-mill)
 - Spin rate evolution depends on the sun elevation history which drives the mean SRP torque (blue) independent of the spin rate while the PMD damping torque (yellow) vanishes with the spin rate
 - Equilibrium spin rate can be estimated by $\omega_z^\infty \sim \frac{\langle T_{SRP} \rangle_{mean}}{M \frac{5}{2} B_{eq}^2 (1 - 0.45 \sin^2 \varphi)} + \frac{9\omega_0}{5 \cos \varphi}$ (orbital magnetic component)



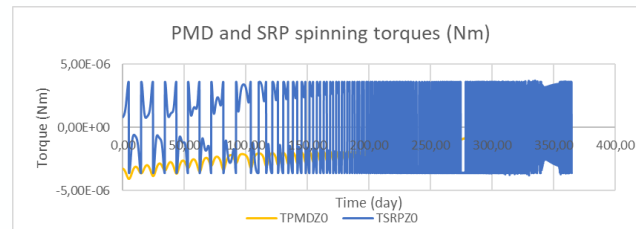
Irregular spin rate due to SRP evolution



5. System level performance validation & verification (1/5)

□ **Necessity of an extensive simulation campaign performed on a dedicated High-Fidelity simulator to assess the PMD (Passive Magnetic Detumbling) performance: “module of angular rates < xx deg/s”**

- 3-axes dynamics and not only spinning phase
- Motivated by the complexity of the SRP torque and low authority of Copernicus Expansion Magnetic Detumbling Systems



Orbit-averaged SRP (blue) and PMD (yellow) spinning torques

- High-Fidelity simulator constraints are different from classical AOCS/GNC simulators
 - No permanent closed loop to mitigate long-term numerical drift of S/C dynamics integration
 - Heavy CPU load and run time for simulations covering several years
 - Combined impact of driving parameters not clearly identified
 - A semi-analytical simulator is recommended, which calculates not anymore the 3-axes instantaneous dynamic evolution but the mid-term and long-term evolution of the angular momentum and spin axis direction
- **ESA is preparing a section on Magnetic Detumbling Performance verification in the SDM Handbook**
- System level verification approach, Hi-Fi simulator development, analytical support, HW verification
 - Examples of main guidelines are presented in the next slides

5. System level performance validation & verification (2/5)

- **Passive Magnetic Detumbling (PMD) performance verification approach (to be initiated early)**
 - a. Performance verification should be performed through numerical simulations on the High-Fidelity simulator
 - b. Dedicated analyses should be undertaken to identify driving parameters
 - ***NOTE 1: Critical parameters in case of lateral Solar Arrays are the Thermo-Optical parameters of front and back faces, their Infrared thermal emission and their misalignments***
 - ***NOTE 2: A semi-analytical simulator is recommended to quickly check the impact upon long term dynamics***
 - c. The selected approach for simulations should be defined and justified
 - ***NOTE 1: Analytical framework can guide the simulation campaign definition***
 - ***NOTE 2: The approach can use for instance Monte Carlo method with 2 sigma confidence level***
 - ***NOTE 3: It is recommended to perform first a series of simulations, focusing on driving parameters affecting the detumbling performance***
 - ***NOTE 4: Simulations should not be stopped as soon as the angular rates fall below a certain value, an apparently successful detumbling can be ruined by spin up/spin down following cycles***
 - d. Analyses should be undertaken to promote a good interpretation of the results
 - ***NOTE 1: Correlation with analytical formulas during the spinning phase is expected: Gravity Gradient precession period, reference time constant, map of SRP spin-averaged torque versus sun elevation, etc.***
 - ***NOTE 2: A detailed interpretation of simulation results is expected to give confidence in long term stabilization of the angular rates***
 - ***NOTE 3: Correlation with semi-analytical simulator results is recommended for cross-validation***

5. System level performance validation & verification (3/5)

□ Passive Magnetic Detumbling (PMD) performance verification approach (to be initiated early)

■ e. Relevant S/C and orbit parameters should be gathered, including at least the following elements:

- Orbital parameters of the spacecraft at mission end of life
- Spacecraft Mass, Centering, Full Inertia Matrix parameters at the end of mission life
- Thermo-optical characteristics of the surfaces in solar and infrared spectrum at EOL
- Estimations or preferably measurements of spacecraft residual magnetic dipole
- Estimation of the overall spacecraft magnetic tensor including Magnetic Detumbling System and Eddy currents,
- Definition of realistic range of parameters (including assumed distribution)

Note: The random generation of parameters should remain realistic and respect fundamental S/C characteristics like major versus minor inertia and principal axes misalignments

■ f. Relevant initial conditions should be defined, covering in particular the following elements:

- Angular rate vector, Obliquity, Local Time of Ascending Node, Epoch, and consequently sun elevation
- Solar Array(s) orientation(s) and other appendages configuration

□ See examples of PMD simulations in Annex B

5. System level performance validation & verification (4/5)

Setup of PMD High Fidelity Simulator (see example in Annex A)

- b. The identification of specific features to be represented or not in the satellite dynamics and the space environment simulation shall be justified and documented with the associated mathematical models.

- **NOTE 1: Rigid body dynamics is generally representative enough for PMD performance assessment.**
- **NOTE 2: 3rd body gravitational interactions are expected to have negligible impact.**

- c. The simulation model of the Magnetic Detumbling System should be validated with respect to the real hardware behaviour characterized in a representative environment.

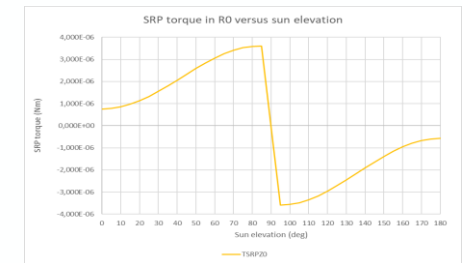
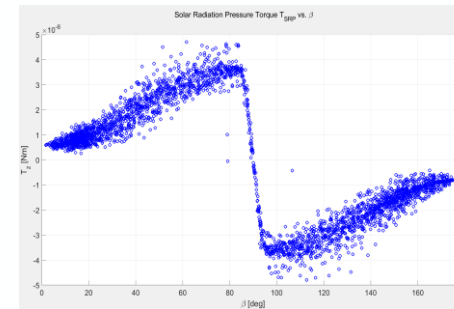
- d. Verification and validation of the entire simulator should be performed using functional test cases.

- **NOTE 1: The generation of orbit-averaged torques to be compared with analytical formulas is a strong contribution for functional validation.**
- **NOTE 2: Cross-validation with a semi-analytical simulator is recommended**

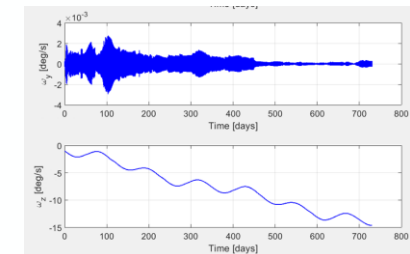
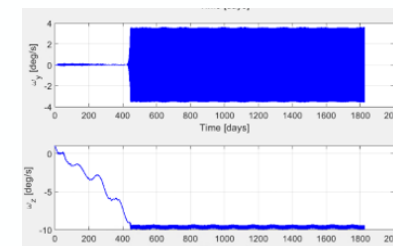
- e. Adequate numerical integration methods and settings for long duration open-loop simulation should be selected.

- **NOTE 1: Artefacts such as fake nutation damping or increase, or sun elevation drift should be eliminated.**
- **NOTE 2: Integration time step size is particularly critical.**

- f. It should be possible to run Monte-Carlo simulations with realistic range of parameters including assumed distributions.



Orbit-averaged SRP spinning torque versus sun elevation
Hi-Fi simulation post processing (left) and analytical model (right)



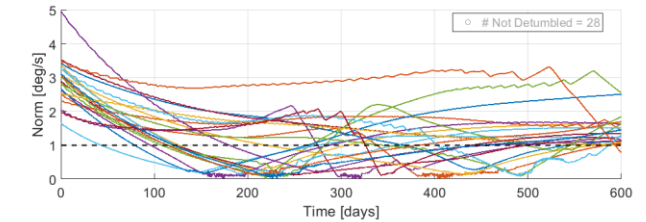
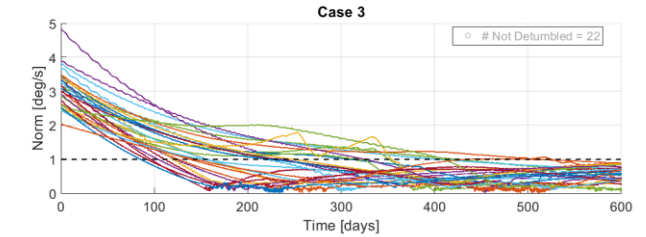
Impact of integration step size: left 10 s and right 1 s (fixed)
Fake nutation increase (left) and correct dynamics (right)

5. System level performance validation & verification (5/5)

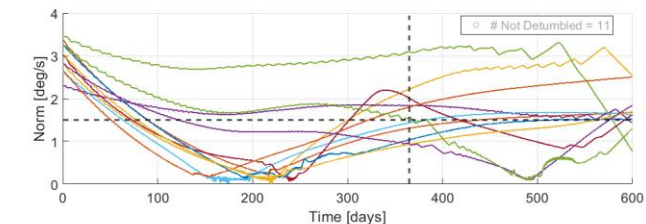
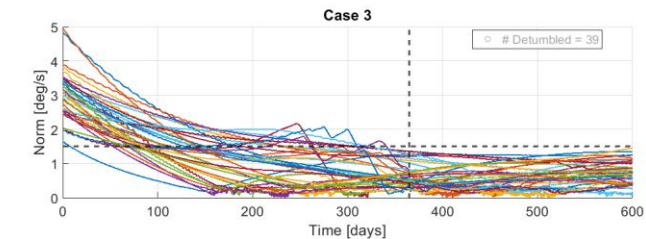
Example of reduced Monte Carlo campaign for a typical SAT LEO

Simulation Parameter	SAT LEO
Orbit altitude (km)	640
Principal Moments of Inertia [I_x, I_y, I_z] (kg. m ²)	[1000.800, 1300] ±5%
Misalignments of major principal axis Z_p wrt geometrical axis Z_G	$ \varepsilon_y \leq 3.5^\circ$ around Y_G and $ \varepsilon_x \leq 3.5^\circ$ around X_G
S/C residual magnetic dipole (A.m ²) (max. values)	[±10, ±10, ±10]
Optimised MTQ Magnetic Tensor ($\Omega^{-1}m^4$)	[7.75, 7.75, 7.75] × 10 ⁴
Eddy currents Magnetic Tensor ($\Omega^{-1}m^4$)	[1.25, 1.25, 1.25] × 10 ⁴
Solar array (SA) area (m ²)	7.0
CoG to SA center of pressure offset (m)	[0, 3.5, 0]

- Solar Array mitigation action => orientation: $\alpha - \varepsilon_y = 3.5^\circ$
- Quick exercise limited to 50 cases, 600 days and variations of:
 - Initial spin rate (between 0 and 5 deg/s), inertia tensor, thermo-optical coefficients
- Success criteria: norm of angular rate vector
 - Norm below 1 deg/s: 44% (22)
 - Norm below 1.5 deg/s : 78% (39)
- Analysis of the results
 - Spin-down phase seems OK except 3 outliers
 - Examine the tumbling phase to gain confidence regarding stabilization
- Conclusion
 - Is relaxed performance acceptable, e.g. 1.5 deg/s instead of 1 deg/s?
 - If not, revisit the Magnetic Detumbling System selection, or Solar Array orientation, or S/C characteristics (during the design and development phase)



44% under 1.0 deg/s



78% under 1.5 deg/s

6. Conclusions

- ❑ **ESA is preparing several satellites for removal in case of failure of the required de-orbiting operations**
 - As part of the Zero Debris approach, ESA aims to generalize preparation for removal to all its future missions
 - The minimisation, prediction and estimation of defunct satellites angular rates are crucial to de-risk removal operations
- ❑ **ESA/Clean Space is preparing a section in the SDM Handbook to provide useful information regarding this activity**
 - A preliminary collection of guidelines has been shared in this presentation
 - At system level, the proposed process is based on High-Fidelity simulations able to perform well thought Monte Carlo campaign and on an analytical framework to correlate simulations, explore driving parameters and guide the simulation campaign
- ❑ **ESA current implementation of the PMD function uses the short-circuiting of Magnetic Torquers at EOL**
 - The test approach designed by ZARM Technik has confirmed the validity of the mathematical models for extremely small induced currents and magnetic dipoles, and validated the MTQ optimisation proposed by ESA and ABSpaceConsulting
- ❑ **Mastering the Long-Term Dynamic Evolution of defunct satellites remains both crucial and complex**
- ❑ **Recommendations for future work:**
 - Maturation of simulators and validation with observation data
 - Consolidation of inputs from satellite design (Eddy currents, Solar Array surfaces optical properties, principal axes misalignments, S/C residual magnetic dipole, etc.)
 - Development of more powerful Passive Detumbling solution
 - Approaches for Solar Array re-orientation in Safe Mode or in case of DNEL (Disconnect of Non-Essential Loads)
- ❑ **Collaboration between stakeholders will be instrumental to progress in all areas**

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7. Acknowledgements and main references

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- “Support to Active Debris Removal Activities in the frame of Copernicus Expansion Missions”, an ESA contract with ABSpaceConsulting, Ref. 4000136689
- “Characterisation and Optimisation of Short-Circuited Magnetic Torquers for Passive Magnetic Detumbling”, an ESA contract with ZARM Technik, Ref. 4000133200.

□ References:

- [RD1] Soares T., Caiazzo A., Wolahan A., Magnetic Damping For Space Vehicles After End-of-life - ESA patent EP19182205 <https://data.epo.org/publication-server/document?iDocId=6430752&iFormat=0>
- [RD2] Benoit A., Ribeiro A., Soares T., Van den Broeck M., Passive magnetic detumbling to enable Active Debris Removal of non-operational satellites in Low Earth Orbit. CEAS Space Journal – April 2021 <https://doi.org/10.1007/s12567-021-00354-8>
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- [RD4] Benoit A., Soares T., Kornienko A., Kowaltschek S., Passive Magnetic Detumbling of Non-operational Satellites in LEO to Enable Active Debris Removal - CleanSpace Industrial Days - September 2021 https://indico.esa.int/event/321/contributions/6396/attachments/4506/6815/Magnetic_Detumbling_Presentation_Clean_Space_Industrial_days_24_Sept_2021.pdf
- [RD5] Benoit A., Soares T., Pereira V., Conings V., Padilla E., Melone E., Imhülse T., Kruse M., Offterdinger P., Validation and Verification of the Long-term Dynamic Evolution of Non-operational Satellites in LEO to Enable Active Debris Removal Missions – 12th International ESA GNC Conference - June 2023 [25f05fc9e20b4a12aec0a2fab9a61d43.pdf \(msecnd.net\)](https://msecnd.net/25f05fc9e20b4a12aec0a2fab9a61d43.pdf)

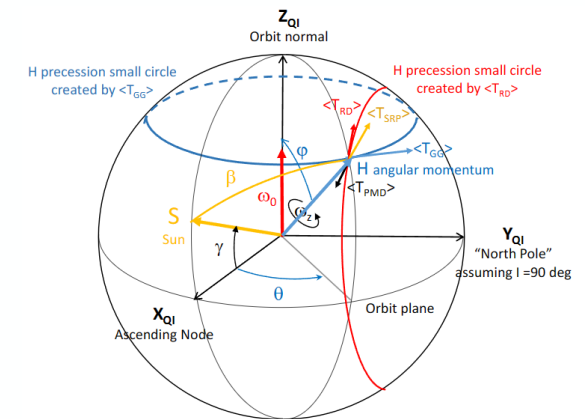
Annex A. PMD simulators examples

ESA/ESTEC High-Fidelity simulator

- 3-axes satellite dynamics and kinematics model
 - Rigid body
 - Gravitational acceleration of Earth, Gravity gradient torque of Earth (no 3rd body)
 - Residual magnetic dipole torque in geomagnetic field
 - Solar radiation pressure torque (Solar Array and body faces), Aerodynamic drag torque
 - Eddy current and Short-circuited magnetic torquers torques in geomagnetic field
- Variable step-size integrator
- Post-processing: spin-averaged, orbit-averaged or day-averaged torques
- Embedded Monte-Carlo functionality

ABSpaceConsulting semi-analytical simulator

- Applicable to Sun-synchronous, circular LEO and spinning phase
 - Long term evolution of spin rate, obliquity and sun elevation
- Implemented in Excel
 - Analytical models of spin-averaged or orbit-averaged torques
 - Gravity Gradient, Residual Dipole, SRP, Passive Magnetic Detumbling
 - Integration time-step between 0.1 and 1 day
 - Immediate results, permitting to explore driving parameters



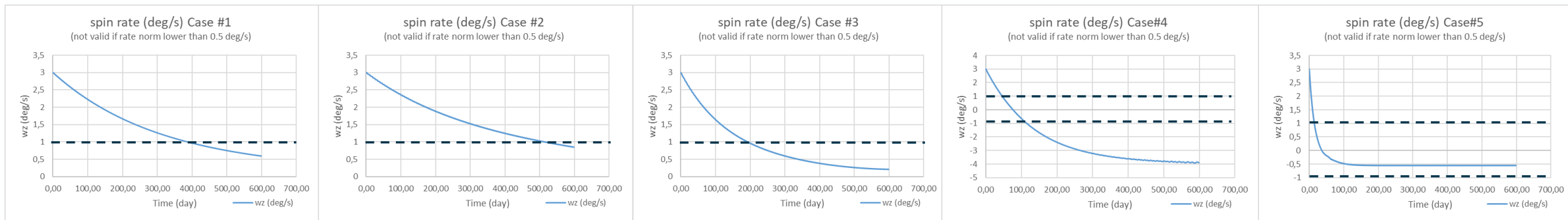
$$\begin{aligned} \dot{H} &= (T_x \cos \theta + T_y \sin \theta) \sin \varphi + T_z \cos \varphi \\ H \dot{\theta} \sin \varphi &= -T_x \sin \theta + T_y \cos \theta \\ H \dot{\varphi} &= (T_x \cos \theta + T_y \sin \theta) \cos \varphi - T_z \sin \varphi \end{aligned}$$

Angular momentum evolution expressed in quasi-inertial reference frame \mathcal{R}_{QI}

Annex B. PMD simulations examples(1/2)

□ Preliminary assessment:

- 5 cases (Magnetic Tensors, Solar Array orientations)
 - Semi-analytical simulations (up) followed by Hi Fi simulations (bottom)



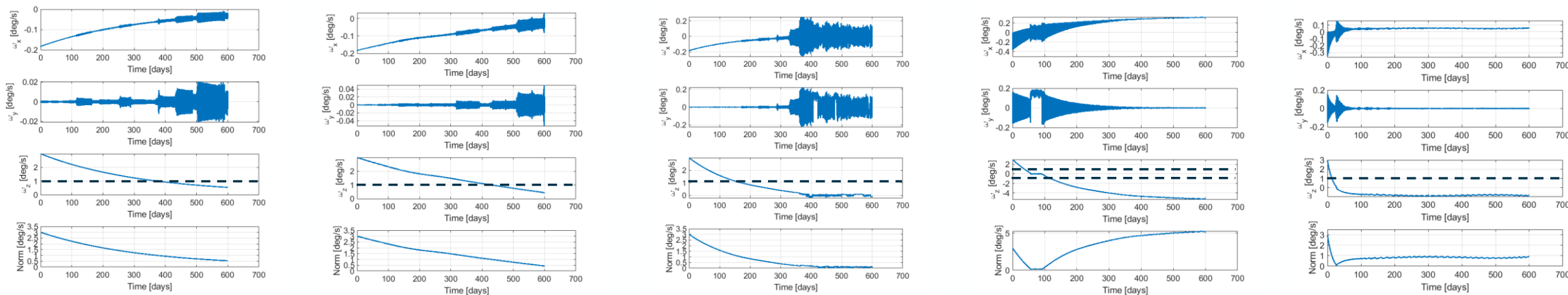
Case #1: M1 and $\alpha - \varepsilon_y = 0^\circ$

Case #2: M1 and $\alpha - \varepsilon_y = 3.5^\circ$

Case #3: M2 and $\alpha - \varepsilon_y = 3.5^\circ$

Case #4: M2 and $\alpha - \varepsilon_y = 90^\circ$

Case #5: M3 and $\alpha - \varepsilon_y = 90^\circ$



- Good correlation (cross-validation), first insight on 3 axes angular rates and follow-up low-rates tumbling phase

Annex B. PMD simulations examples (2/2)

- Case #3 is baselined: optimised MTQ and Solar Array orientation close to the major principal axis
 - Large exploration of driving parameters with semi-analytical simulator: initial obliquity, S/C magnetic moment, thermo-optical parameters and InfraRed reemission
 - Insight upon SRP/PMD competition, guess possible final evolutions of sun elevation, SRP torques and spin rate

□ Monte Carlo campaign

- Selection of input data and statistics, guided and justified by previous assessment and analyses
 - Pay attention that the random generation of parameters remains realistic and respects important S/C characteristics like major versus minor inertia and principal axes misalignments
- Performance with statistical distribution, analysis of the results and conclusion
- If necessary, consider acceptance of relaxed performance or revisit the Magnetic Detumbling System selection, or Solar Array orientations, or S/C characteristics

□ See reduced Monte Carlo campaign on slide 15

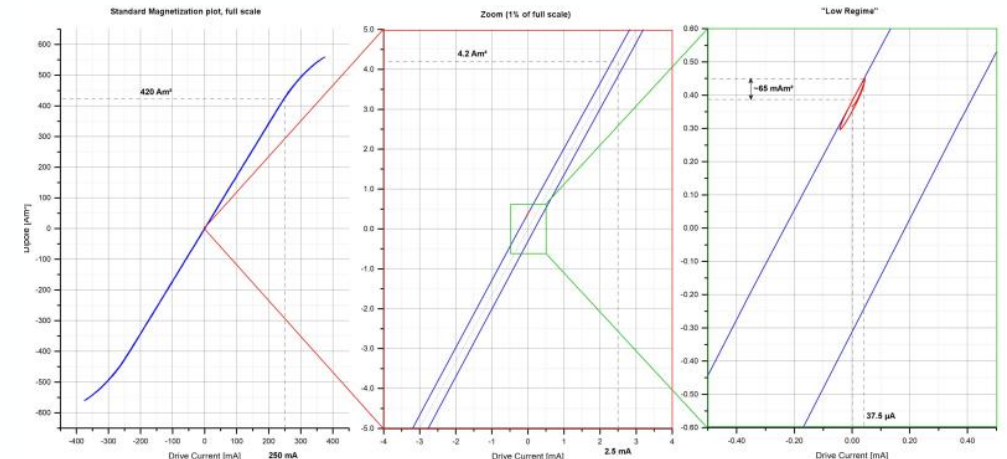
Annex C. HW level verification of a PMD system (1/2)

- **System analyses and simulations rely on mathematical models of the PMD System(s)**
 - Models and parameters need to be verified by HW tests in a representative environment
 - energy dissipation, induced or Eddy currents, magnetic tensor, magnetic moment and torques ...
 - For short-circuited Magnetic Torquers (MTQs) immersed in a rotating magnetic field

- $i = - \frac{\mu_{rod} N_{turn} A_{core}}{R} \left(\frac{d\vec{B}_{Earth}}{dt} \right)_{sat} \cdot \vec{X}_k$ and $M = \frac{2 \mu_{rod}^2 N_{turn}^2 A_{core}^2}{R} = \frac{2 L^2}{\mu_0^2 R} \left(\frac{l_{rod}}{N_{turn}} \right)^2 = \frac{2 \mathcal{M}(i)^2}{R i^2}$, and other formulas
- Induced currents in short-circuited coils are around 100 μA instead of 100 mA in operational regime
- It was crucial to verify the behaviour in low regime

- **ZARM Technik, supplier of Copernicus Expansion MTQs, confirmed the magnetic core excitation in low regime**

- The test within a rotating field 100 μT at 3 $^\circ/s$ confirmed small hysteresis loops keeping nominal slope $\frac{\mathcal{M}(i)}{i}$
- If not, the MTQ would create induced currents as a simple air-coil without magnetic core ($\mu_{rod} = 1$ and not 300) and its magnetic tensor would vanish in the ratio $\sim 90\,000$

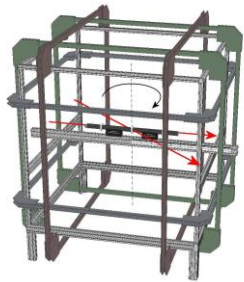


2-step zooming of Magnetic Moment \mathcal{M} versus Current i : the magnetic core is correctly excited at low regime as shown by the slope of the small hysteresis loop
ZARM Technik

Annex C. HW level verification of a PMD system (2/2)

Conventional methods were adapted to measure the expected low inductions

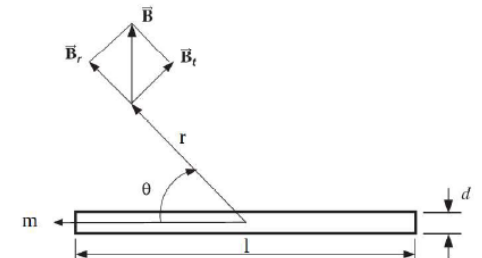
- A dedicated test approach was designed by ZARM Technik
 - The very low induced current is measured in a 3D Helmholtz coil generating the representative rotating field
 - This very low sine current is applied in a precise laboratory test set up, the magnetic dipole components created B_r and B_t are measured and magnetic moments \mathcal{M} are derived from analytical formulas



*Helmholtz coil generating representative rotating field (with Earth field compensation)
Credit ZARM Technik*

MT400-2-D21071301	B_{test} (μT)	ω ($^\circ/s$)	i (μA)	\mathcal{M} (Am^2)	\mathcal{M}/i	$M \pm \Delta M$
Single coil	100	6	159 (166.4)	0.488 (0.502)	3069	93210 ± 2980 (95100)
Single coil	50	6	79 (83.2)	0.242 (0.251)	3063	92620 ± 3060 (95100)
Coils connected in parallel	100	6	318 (332.8)	0.976 (1.004)	3069	93210 ± 2940 (95100)
Coils connected in parallel	50	6	159 (166.4)	0.488 (0.502)	3069	93210 ± 2980 (95100)
Coils connected in parallel	100	3	160 (166.4)	0.491 (0.502)	3069	93790 ± 3000 (95100)

Some test results (the predictions of mathematical models are between brackets)



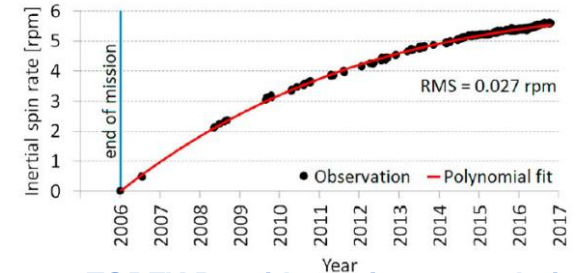
*Magnetic moment \mathcal{M} is derived from the measurement of B_r and B_t
Credit ZARM Technik*

- The slope \mathcal{M}/i is constant and the test results confirm the validity of the mathematical models
- ZARM has tested 14 Magnetic Torquers. The prototype above was optimized for PMD by maximizing the electrical time constant and replacing Al housing by non-conductive CFRP housing
- The magnetic tensor during PMD will be larger due to lower coil resistance at very low temperature

Annex D. TOPEX Poseidon tentative analysis (1/5)

TOPEX/Poseidon is one important reference case, having shown that large angular rates can appear after a successful decommissioning.

- Several papers ([RD6], [RD7], [RD9]) have successfully reproduced the spin rate evolution by High-Fidelity simulations.
- Such identification exercise is however delicate and not fully reliable, especially when important driving parameters like S/C inertias and Solar Array orientation are not known.
- A tentative analytical interpretation is proposed



TOPEX Poseidon spin rate evolution
Credit [RD6]

The spin rate reconstruction on slide 4 shows meaningful features:

- a) a remarkably regular and smooth evolution, showing a characteristic exponential pattern.
- b) an asymptotic limit for the angular rate ω_s^∞
- This looks like the signature of a permanent spinning torque, in competition with a damping torque created by Eddy currents. Such pattern is easily modelled by a very simple macroscopic equation:

$$\langle T_{SRP} \rangle_{long\ term} + \langle T_{PMD} \rangle_{long\ term} = I_s \dot{\omega}_s$$

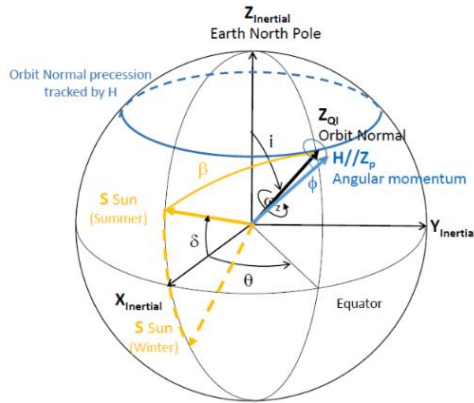
Passive Magnetic Detumbling torque in polar orbit can be extrapolated to an inclined orbit by recomputing:

$$\langle \overrightarrow{T_{SC1D}} \rangle_{orbit} = -MB_{eq}^2 \vec{\omega}_z \frac{1}{T_{orbit}} \int_0^{T_{orbit}} \|\vec{B}_\perp\|^2 dt$$

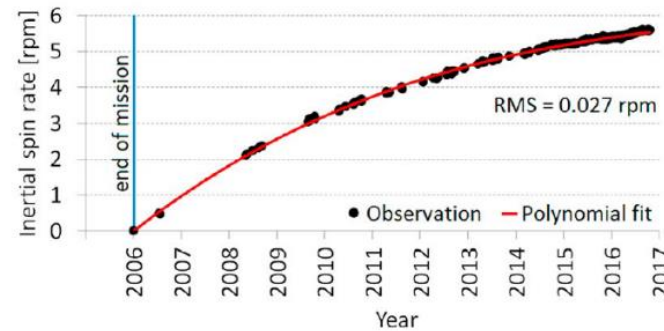
$$\langle T_{SRP} \rangle_{long\ term} - MB_{eq}^2 \frac{5}{2} \left[\sin^2 i - 0.5 \sin^2 \varphi \left(1 - \frac{9}{5} \cos^2 i \right) \right] \omega_s = I_s \dot{\omega}_s$$

Annex D. TOPEX Poseidon tentative analysis (2/5)

- Assuming that the obliquity is close to zero (spin axis parallel to the orbit normal) and that the mean SRP torque is reasonably constant, the spin rate evolution would be: $\omega_S = (\omega_{S0} - \omega_S^\infty)e^{-t/\tau} + \omega_S^\infty$ with $\tau = \frac{I_s}{2.10 MB_{eq}^2}$ and $\omega_S^\infty = \frac{\langle T_{SRP} \rangle_{long\ term}}{2.10 MB_{eq}^2}$
 - With these assumptions, the dynamic evolution of the angular momentum could be as shown hereunder with the angular momentum tracking the precession of the orbit pole in a spiraling motion.
 - It is easy to find parameters matching the reconstructed spin history



Possible dynamic motion of TOPEX/Poseidon

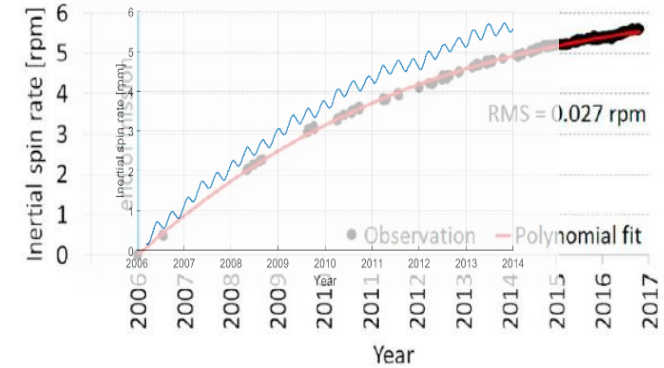


Parameter fitting: $\omega_S = (\omega_{S0} - 39^\circ/s)e^{-t/5.8\ years} + 39^\circ/s$

- TOPEX/Poseidon inertias are apparently not known from recent authors, but they are reported in Fig. 3 of [RD8]:
 - $I_x = 6912\ \text{slug-ft}^2 = 9371\ \text{kgm}^2$; $I_y = 3107\ \text{slug-ft}^2 = 4213\ \text{kgm}^2$ and $I_z = 8604\ \text{slug-ft}^2 = 11665\ \text{kgm}^2$
- Assuming that TOPEX/Poseidon is spinning around its major principal axis
 - the magnetic tensor of Eddy currents can be derived: $M = \frac{I_z}{2.10 B_{eq}^2 \tau} = 10.5 \times 10^4\ \Omega^{-1}\text{m}^4$
 - The value of the mean SRP torque is given by: $\langle T_{SRP} \rangle_{long\ term} = 39 \frac{\pi}{180} 2.10 MB_{eq}^2 = 4.0 \times 10^{-5}\ \text{Nm}$

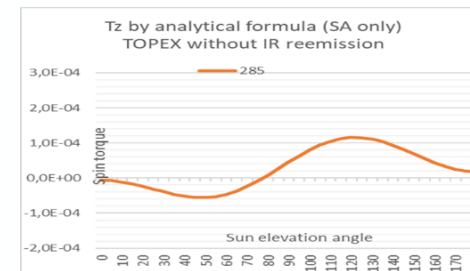
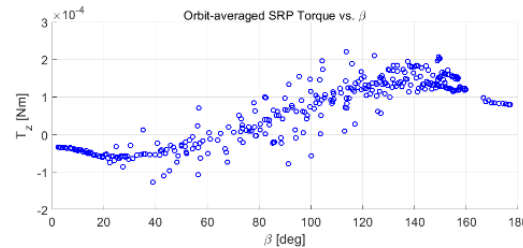
Annex D. TOPEX Poseidon tentative analysis (3/5)

- A simulation has been performed with ESA/ESTEC High-Fidelity simulator.
 - The Solar Array orientation has been taken from [RD6] at 285 deg (75 deg with our convention), the magnetic tensor of Eddy currents at $10.5 \times 10^4 \Omega^{-1}m^4$ and the reliable inertias taken from [RD8]
 - The initial obliquity was set at zero, the initial spin rate at 1 deg/s around the major principal axis.



Hi-Fi simulation does not perfectly match in orbit observation

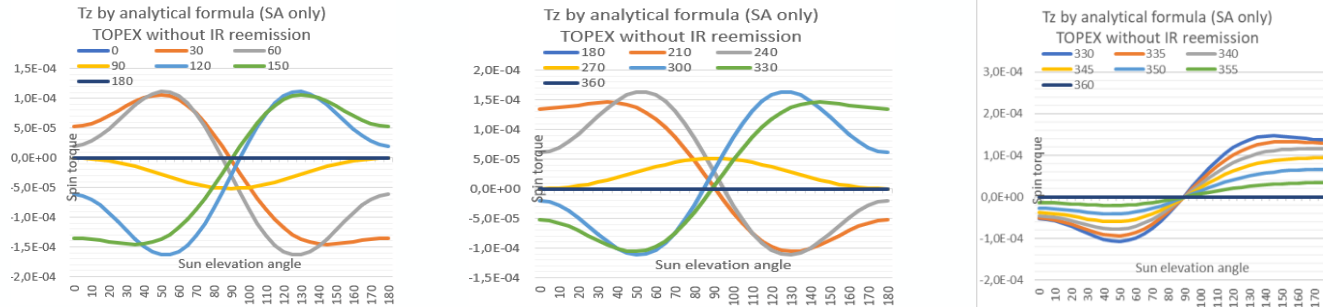
- Matching is not perfect: either the detumbling torque of Eddy currents is less efficient or the SRP spinning torque generated by the simulator is too high due to the complex evolution of the sun elevation.
- An interesting post-processing allows to validate the analytical map SRP torque versus sun elevation for this Solar Array orientation as shown hereunder



SRP orbit-averaged torque post-processing confirms the analytical model map

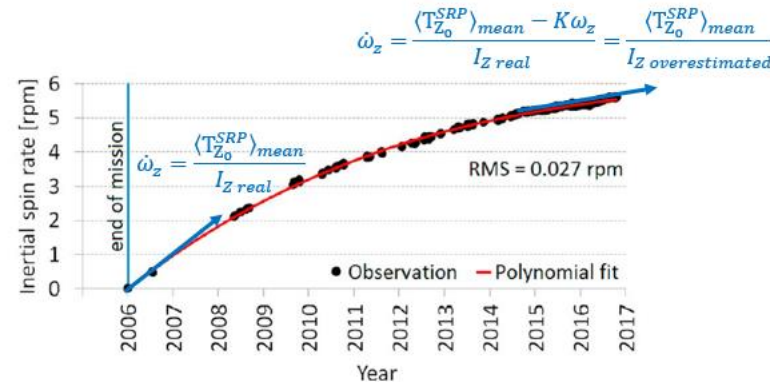
Annex D. TOPEX Poseidon tentative analysis (4/5)

- This was a priori far from obvious, looking at the diversity of the analytical maps shown hereunder for a variety of Solar Array orientations.



Spinning torque versus sun elevation angle for various Solar Array orientations

- Precursor simulations in [RD7] did not model Eddy currents damping torque and estimated a spin inertia as large as 70 000 kg m2. This large virtual value corresponds indeed to the slope of the spin rate evolution if ignoring the Eddy current damping term as visualized hereunder



Ignoring Eddy currents damping, the spin inertia is overestimated in [RD7]

Annex D. TOPEX Poseidon tentative analysis (5/5)

□ References

- [RD6] Kucharski, D. et al: Spin-up of space debris caused by solar radiation pressure (2017)
Proc. 7th European Conference on Space Debris, Darmstadt, Germany, 18–21 April 2017,
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- [RD8] Dennehy, C. J.: Attitude Determination and Control Subsystem for the TOPEX Satellite
<https://arc.aiaa.org/doi/abs/10.2514/6.1988-4129>
- [RD9] Sagnières, L.: Investigation into the rotational dynamics of the defunct satellite TOPEX/Poseidon
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