Geant4 Low Energy Rayleigh Scattering

- Modification of the polar angle sampling
- Introduce the polarization processes:

+ Sampling method of the azimuthal angle.

+ Polarization direction of the scattered radiation.

Modification of the polar angle sampling:

Motivation → Error in the angular distribution at small angles (problem report #371 and #406)

Solution → Change the sampling method

Cross section:

The Thomson cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_T = r_0^2 \cos^2\Theta$$

For unpolarized photons, the Rayleigh cross section is:

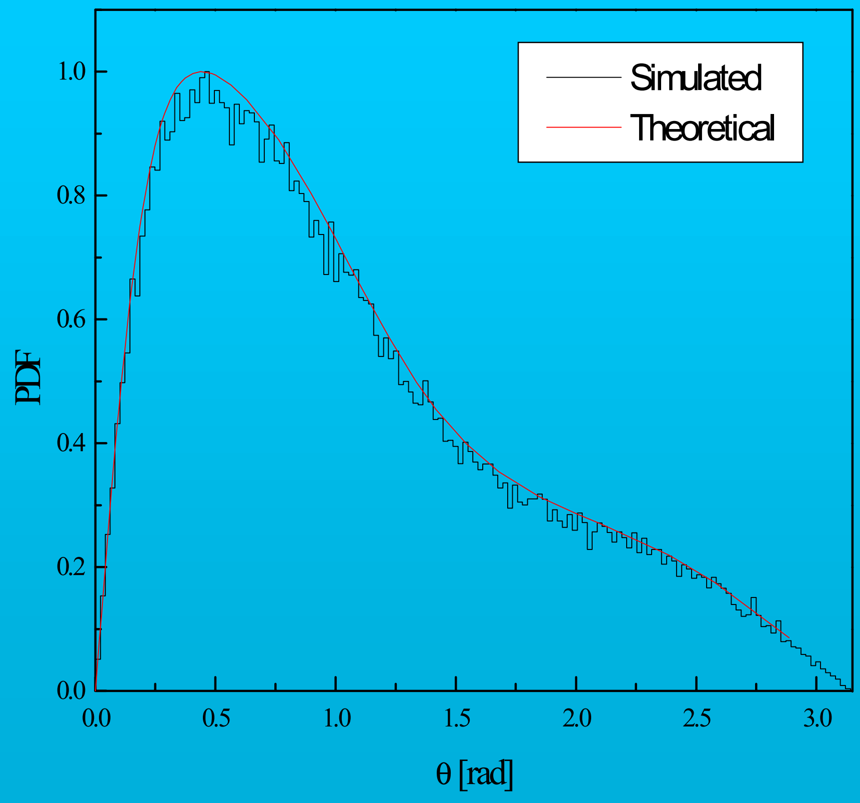
$$\left(\frac{d\sigma}{d\Omega}\right)_{R,U} = r_0^2\, F^2(x,Z)\left[\frac{1}{2}(1+\cos^2\theta)\right]$$

Reject function

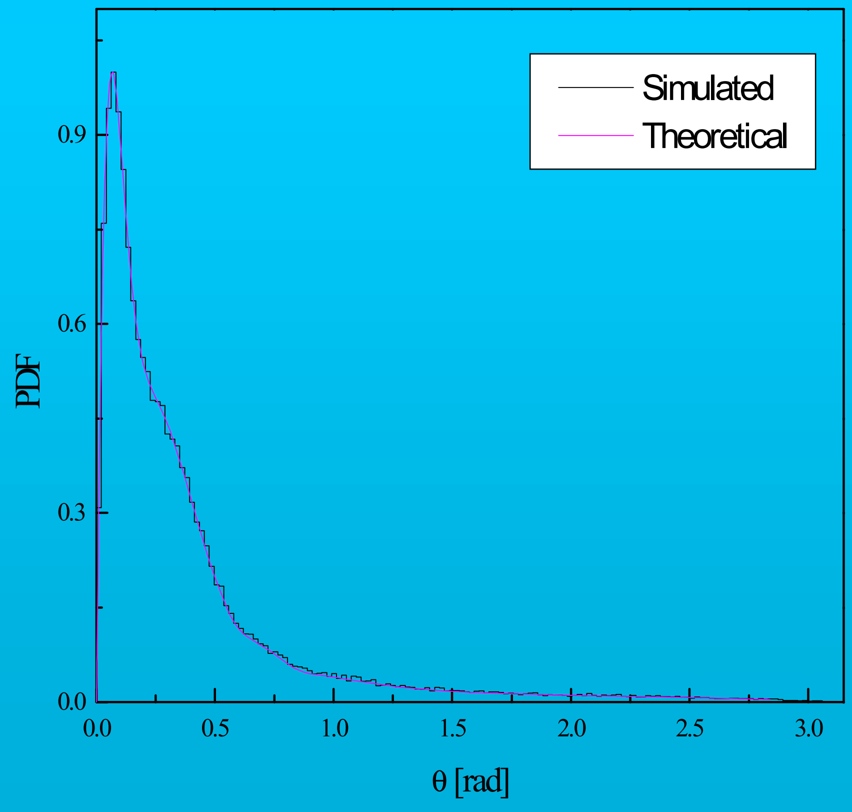
PDF to sample cos θ

Examples of the angular distribution

Si at 10 keV



Fe at 100 keV



Polarization processes:

- Sampling method of the azimuthal angle.

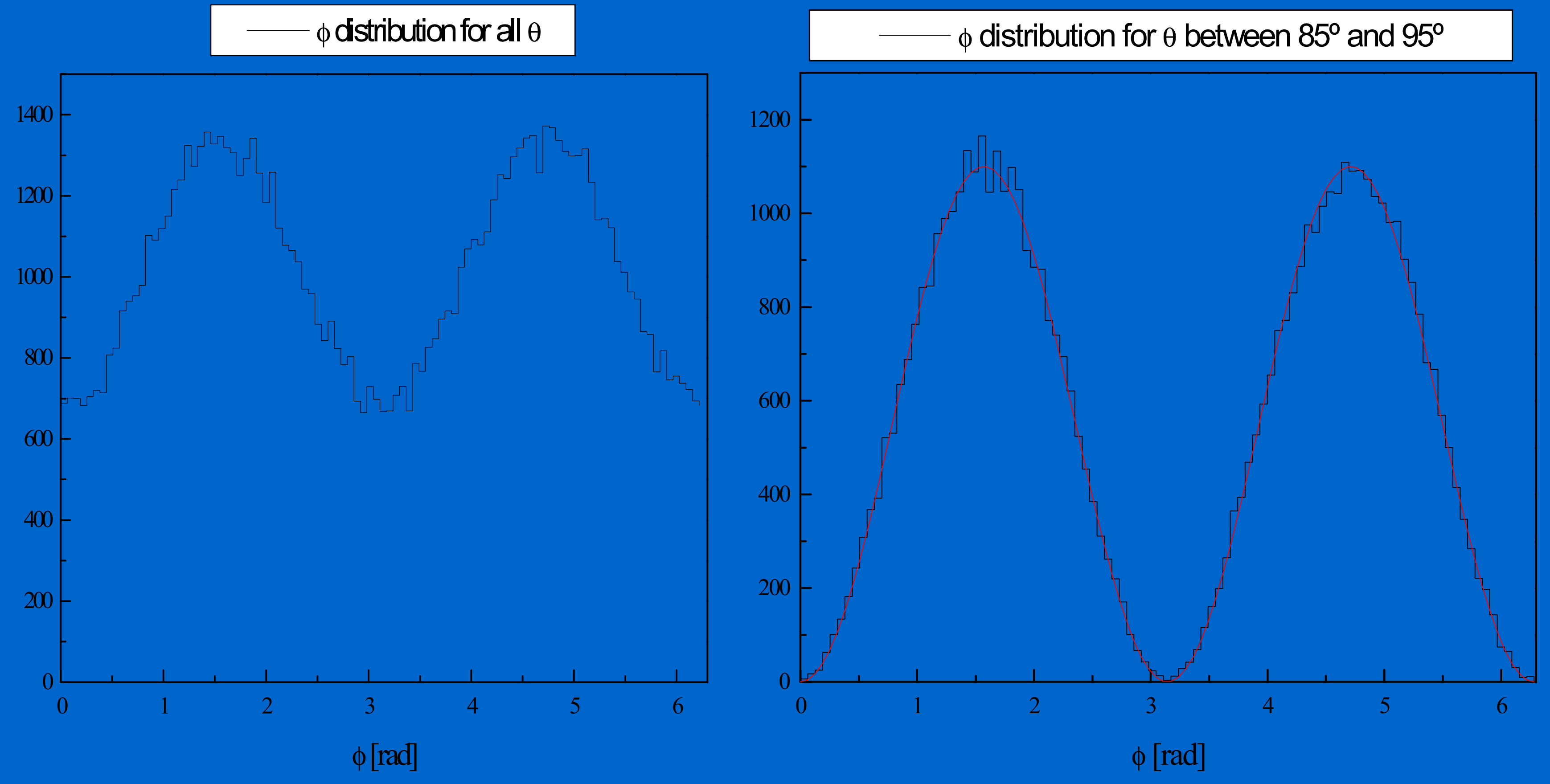
The cross section for the polarized case can be obtained from the Stokes parameters. The matrix of this process is the Comtom matrix in the limit case: $k = k_0 \longrightarrow 0$. Adopting the same philosophy as Compton for the binding effect (this is, F has no dependence on the polarization), we obtain:

$$\frac{d\sigma}{d\Omega} = r_0^2 \; F^2(x, Z)(1 - \sin^2\theta \cos^2\phi)$$

Where: $x = \frac{1}{\lambda[\overset{o}{A}]} \sin(\theta / 2)$

This cross section is similar to the Compton case => the same procedure to sample ϕ

Example of the azimuthal distribution at 5 keV in Si:



•Vector Polarization:

Since the calculations for the Compton effect were done using only geometrical consideration => are valid to describe the direction of the vector polarization of the scattered radiation for the Rayleigh.

Using the Stokes parameters:

√ Unpolarized Radiation

Degree of Polarization:

$$P = \frac{\sin^2\theta}{1+\cos^2\theta}$$

Ratio of intensities:

$$p = \frac{1}{\cos^2\theta}$$

√ Polarized Radiation

Degree of Polarization:

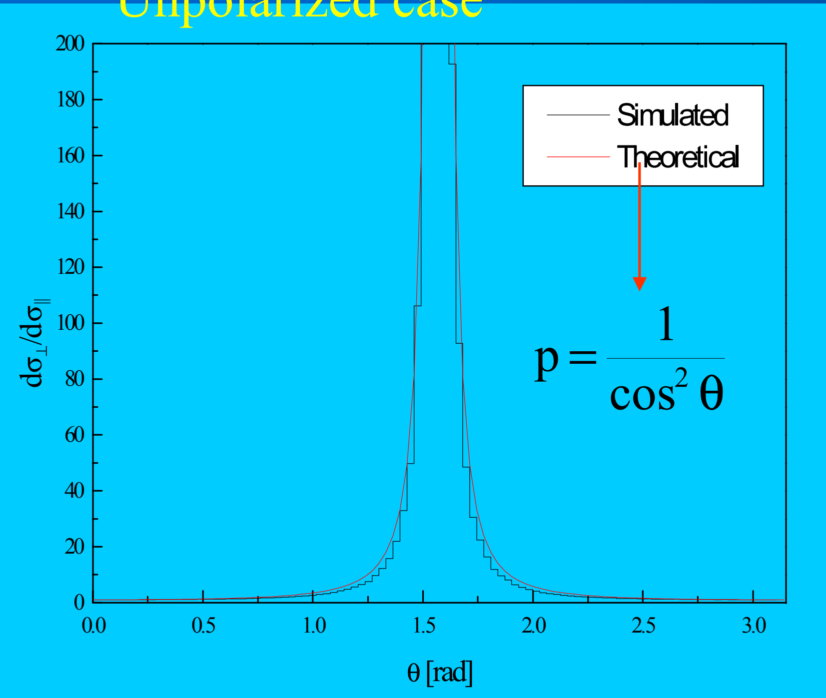
$$P = 1$$

Ratio of intensities:

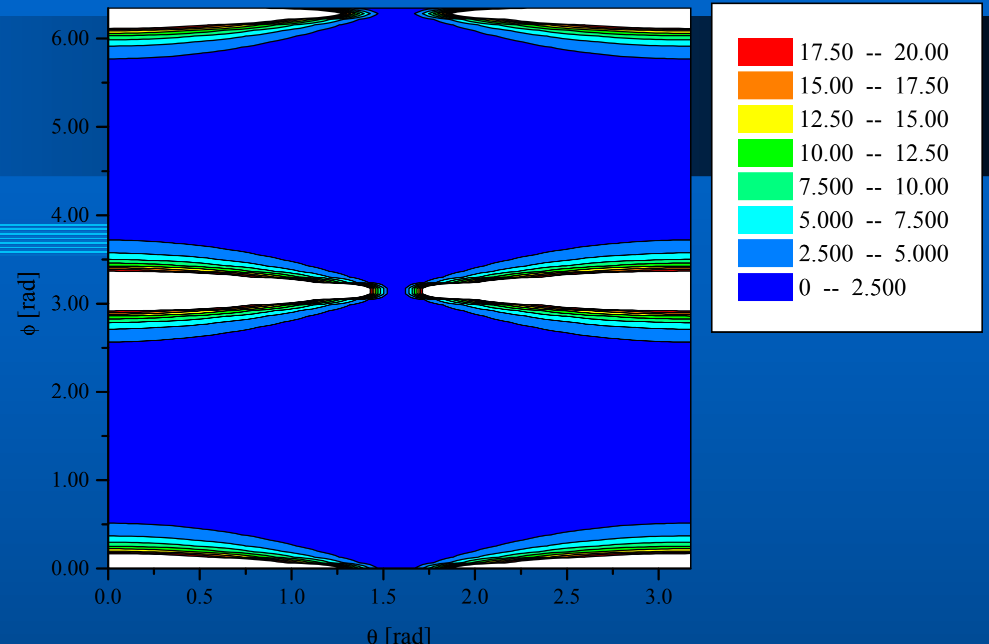
$$p = \frac{1}{\cos^2\theta\tan^2\varphi}$$

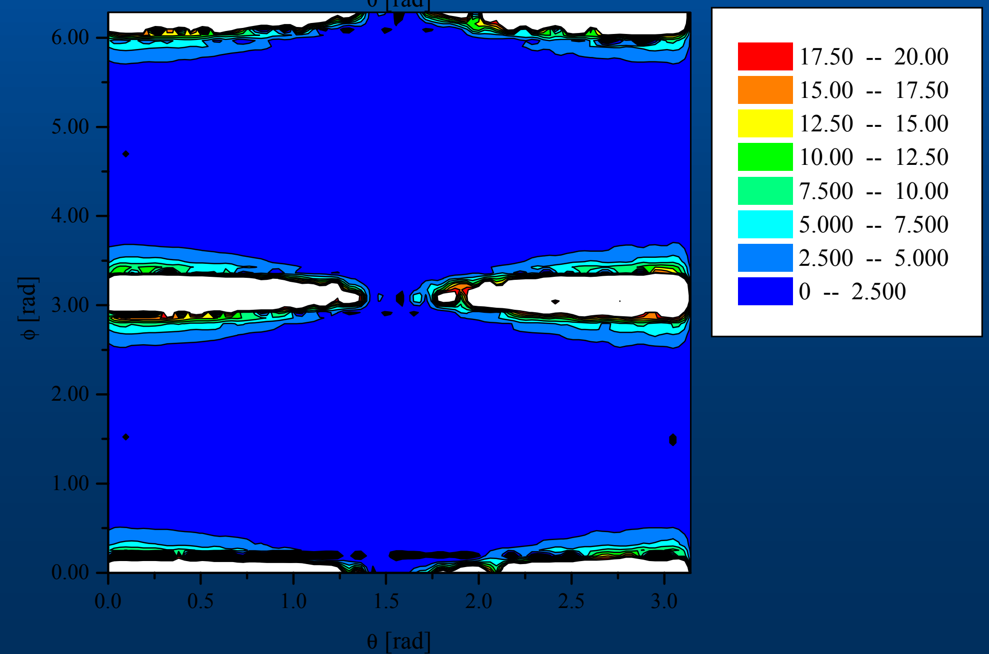
Intensity ratios

Unpolarized case



Polarized case: upper: theoretical, down: monte carlo.





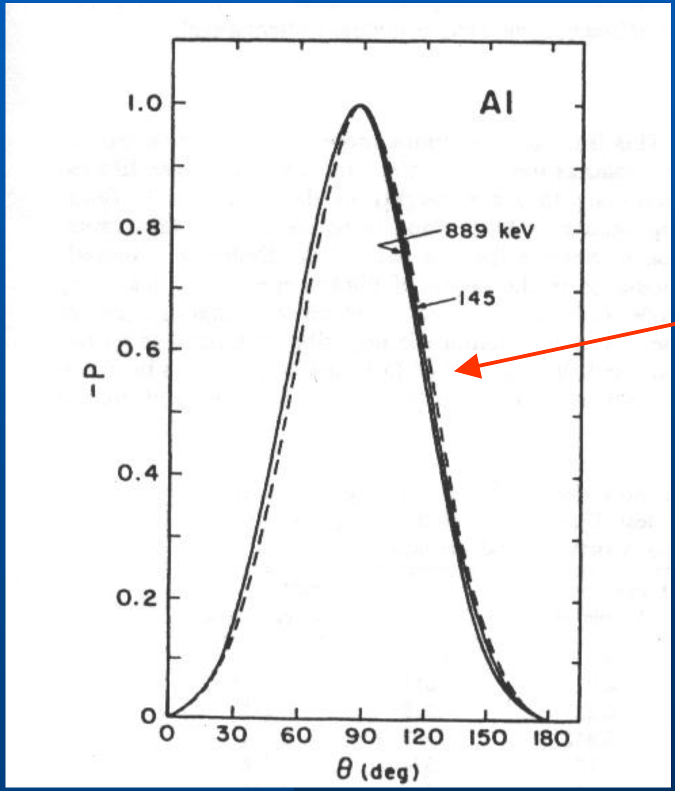
Comparison of intensity ratios between the theoretical prediction and monte carlo simulation for 5 keV incoming photons. For the polarization case, the graph show the surface level for the inverse of p =>

$$p^{-1} = \cos^2\theta \tan^2(\pi/2 - \phi)$$

Binding Effects

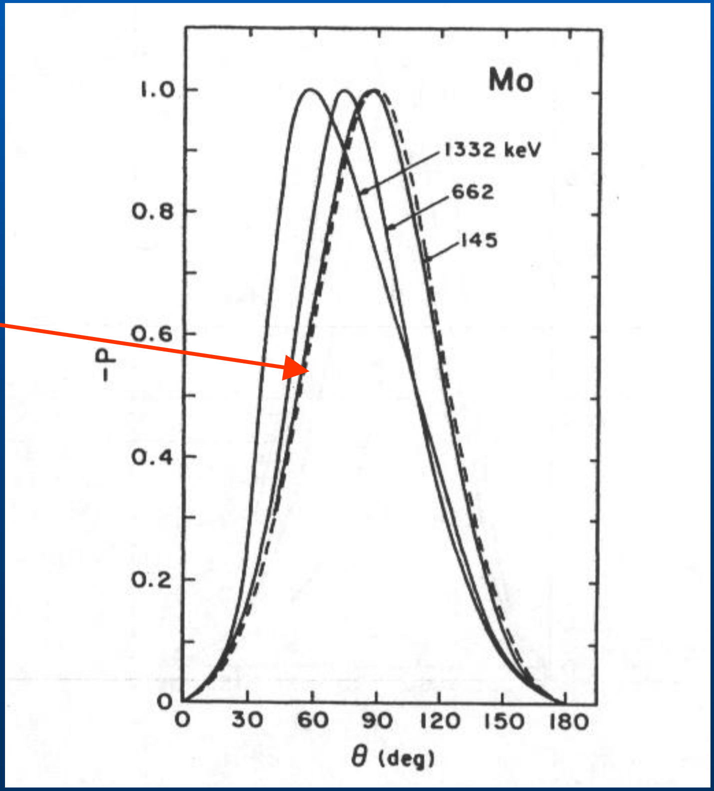
In our calculations we assume that the form factor has no dependency with the polarization, however there is an experimental and theoretical evidence that this is not completely correct (S. C. Roy et al. Phys. Rev. A 34, 1178 (1986)):

Theoretical expectation of the influence of the polarization in the degree of polarization



Dashed line

$$P = \frac{\sin^2\theta}{1+\cos^2\theta}$$



Experimental evidence

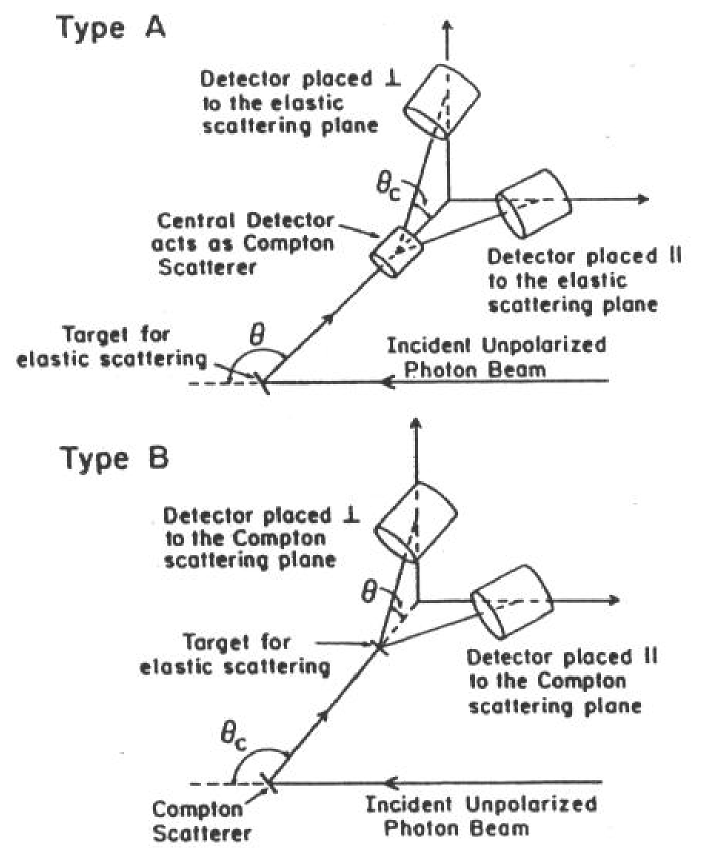


FIG. 1. Schematic diagram of two types of polarization experiments performed in the energy region of our interest.

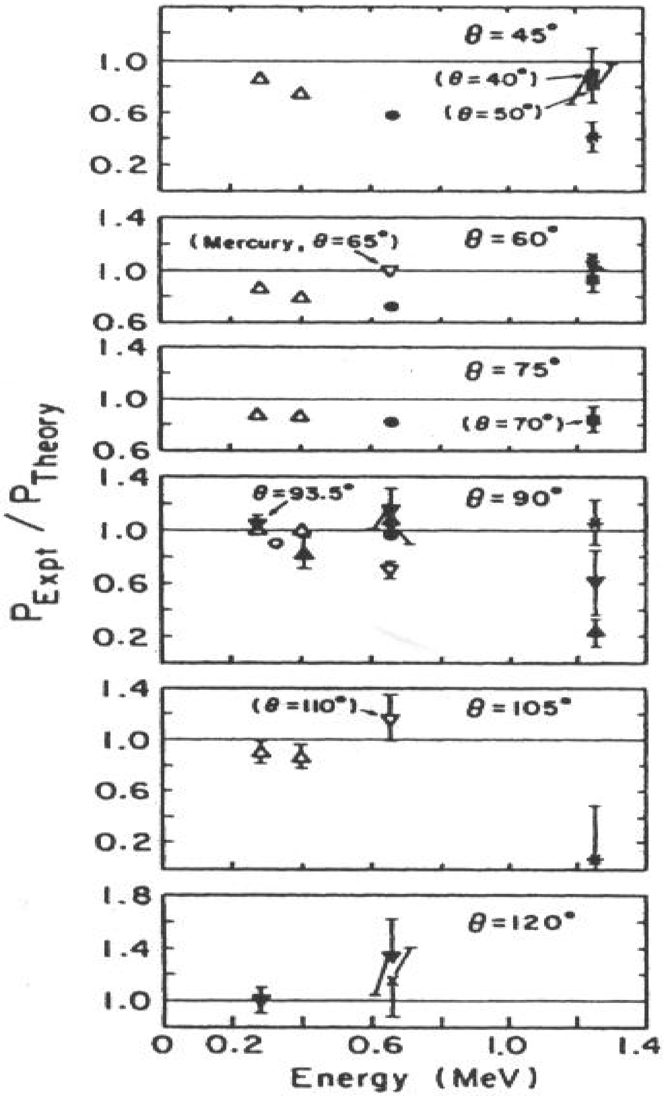


FIG. 6. Comparison of experimental polarization values with theoretical values, shown as the ratio of experimental to theoretical values (P_{expt}/P_{theory}) for lead at different photon energies and different angles of scattering. Open symbols represent experiments of type B (see text for details). ▲, Ref. 17; ×, Ref. 18; ▼ Ref. 19; ●, Ref. 20; ∗, Ref. 22; ■, Ref. 23; △, Ref. 24; ○, Ref. 25; ▽, Ref. 26.