

# Attitude Estimation of Inactive Resident Space Objects from Photometric Measurements Using Particle Filtering

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40<sup>TH</sup>

Excellence and beyond

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FOUR DECADES **PUSHING THE LIMITS**

# Agenda

- Introduction
- Attitude estimation from photometric measurements
- Particle filtering methods for attitude estimation
- Results
  - ✓ Case 1: unimodal posterior PDF
  - ✓ Case 2: multimodal posterior PDF
- Conclusions and future work

# Introduction

# Introduction

## ■ Resident Space Objects (RSOs) characterisation

- New Space era → space traffic ↑ space debris ↑
- Mitigate the proliferation of space debris
- Holistic characterisation of RSOs
- Estimate size, shape, materials, rotation, attitude...
- Use ground-based sensors (telescopes, radars) for observations

## ■ Applications in the context of SST & SEP

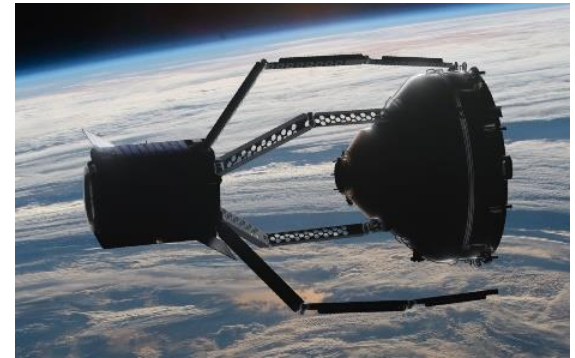
- Mission analysis and design
  - Active Debris Removal (ADR)
  - In-Orbit Servicing (IOS)
- Address uncontrolled re-entries
- Verify contingency attitude modes



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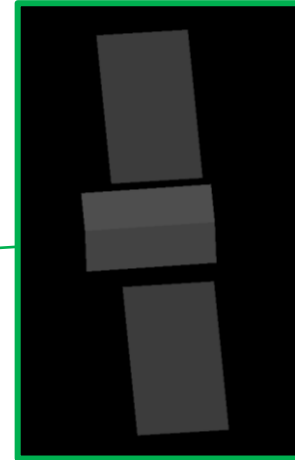
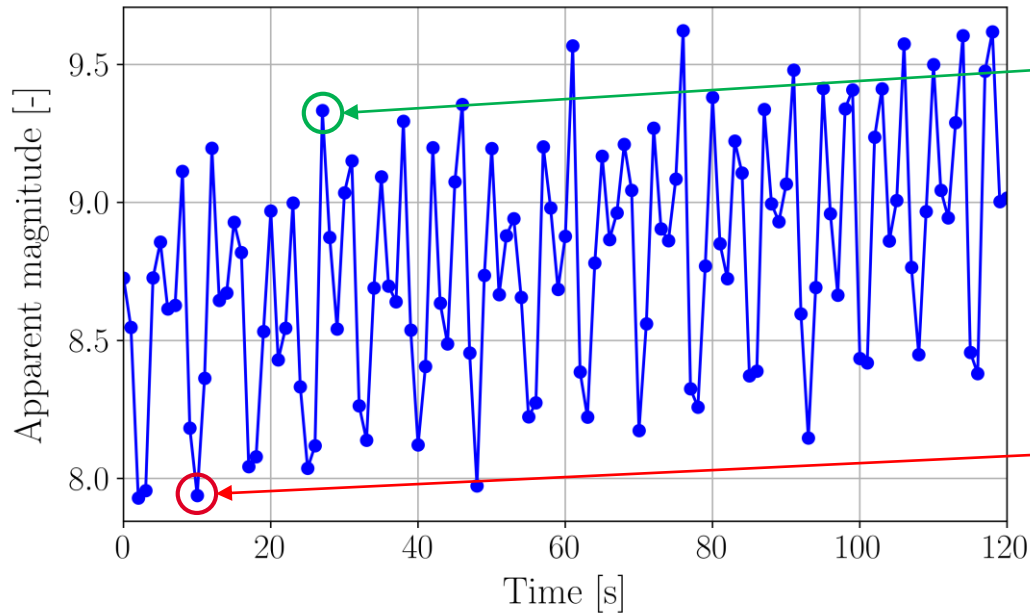
# Attitude estimation from photometric measurements

# Attitude estimation from photometric measurements

## ■ Light curves

- Photometric measurements obtained with ground-based telescopes

- Apparent magnitude:  $m = -2.5 \log\left(\frac{I_o}{I_{ref}}\right)$



# Attitude estimation from photometric measurements

## ■ Typical assumptions in light curve inversion for attitude estimation

- Knowledge of the RSO's shape, size and surface optical properties
- Knowledge of the Aerosol Optical Depth (AOD)

## ■ Challenges of the light curve inversion problem

- Ambiguities in measurements
- Nonlinear measurement model

## ■ GMV's previous experience

- Least Squared Method (LSM)
- Unscented Kalman Filter (UKF)
  - Reliance on an initial estimate
  - Tendency to converge to local minima



# Particle filtering methods for attitude estimation

# Particle filtering methods for attitude estimation

## ■ Particle filter

- Approximate Bayesian estimator that represents probability densities using a weighted set of samples
- It effectively handles multimodal probability density functions (PDFs)

## ■ Formulation

- The posterior PDF at time  $k$  is approximated by:

$$p(\mathbf{x}_k | \mathbf{z}_{1:k}) \approx \sum_{i=1}^{N_s} w_k^i \delta(\mathbf{x}_k - \mathbf{x}_k^i)$$

- Principle of importance sampling  $\longrightarrow$  weights. Samples are drawn from a proposal density  $q$ .

$$w_k^i \propto w_{k-1}^i \frac{p(\mathbf{z}_k | \mathbf{x}_k^i) p(\mathbf{x}_k^i | \mathbf{x}_{k-1}^i)}{q(\mathbf{x}_k^i | \mathbf{x}_{k-1}^i, \mathbf{z}_k)}$$

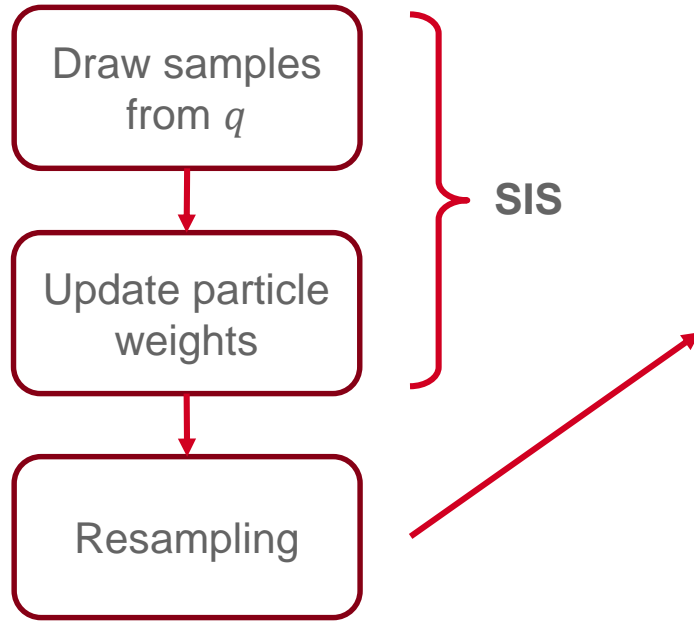
- If the importance density is chosen to be the prior:

$$q(\mathbf{x}_k^i | \mathbf{x}_{k-1}^i, \mathbf{z}_k) = p(\mathbf{x}_k^i | \mathbf{x}_{k-1}^i) \longrightarrow w_k^i \propto w_{k-1}^i p(\mathbf{z}_k | \mathbf{x}_k^i)$$

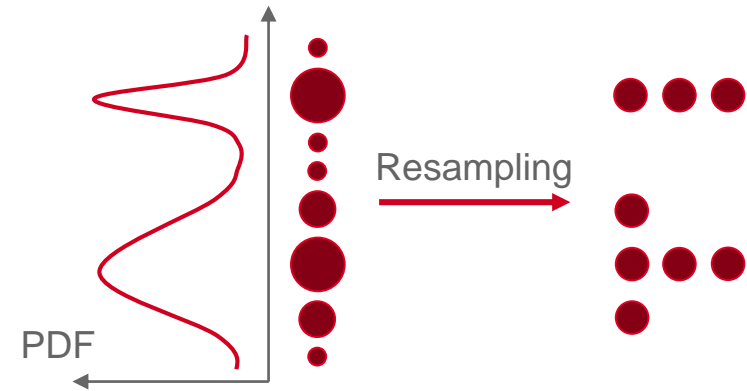
# Particle filtering methods for attitude estimation

## ■ Generic particle filter

- Sequential Importance Sampling (SIS) algorithm + Resampling



Resampling avoids the degeneracy problem...

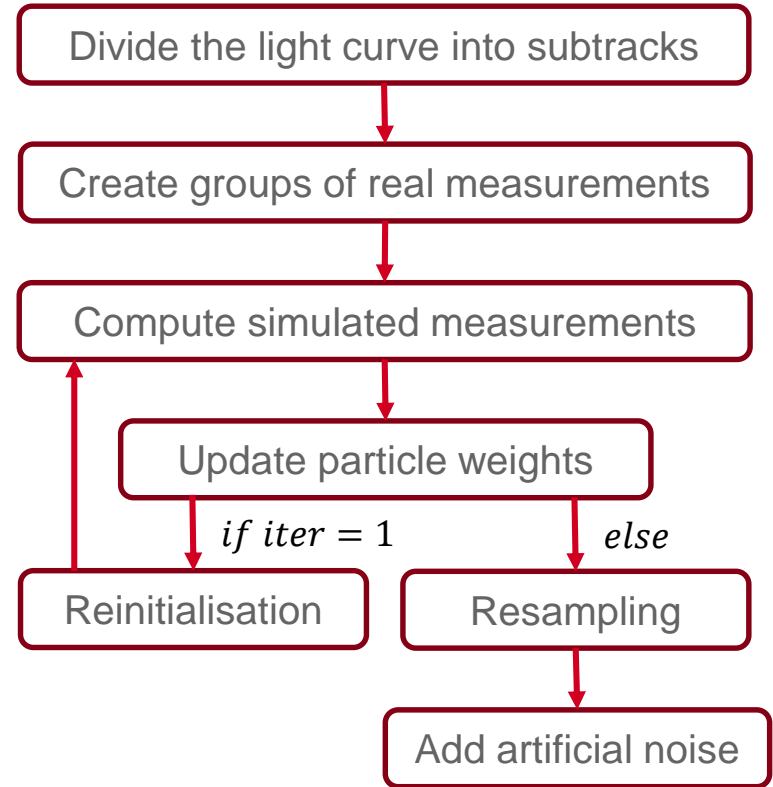


...but may produce **sample impoverishment**

# Particle filtering methods for attitude estimation

## ■ Developed particle filter

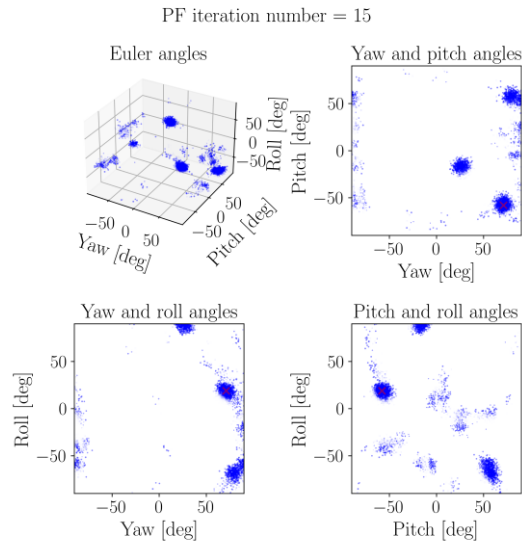
- Estimates the attitude of the RSO at a specific time
- **Pseudo-batch algorithm**
  - Selects measurements from light curve subtracks
  - Updates particle weights according to the RMSE
  - Advantage: reduces the occurrence of local minima
- **Reinitialisation** in the first iteration
  - Moves particles with higher RMSE to regions of higher probability
  - Recomputes the weights of the new particles
  - Advantage: refines the initial posterior PDF



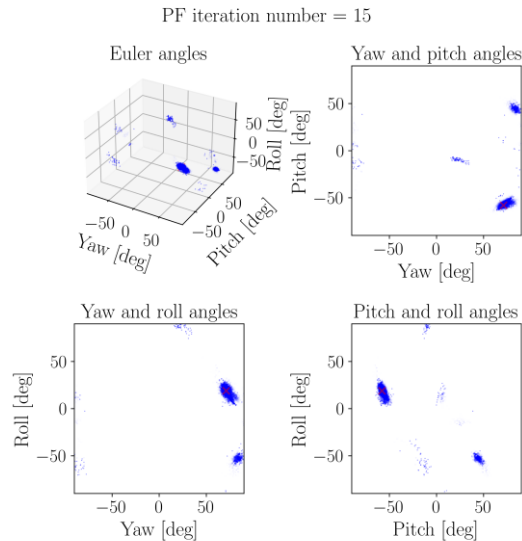
# Particle filtering methods for attitude estimation

## ■ Generic vs. developed particle filters

### Generic (sequential) PF



### Pseudo-batch PF



## Advantages

Less local minima

Higher accuracy

Better computational performance

Noisy real measurements

Very high flexibility

# Particle filtering methods for attitude estimation

## ■ High-fidelity simulator of photometric observations

- GMV's **Grial** tool, implemented in Java using OpenGL.
- Calculates contributions to reflected light from each illuminated and visible pixel on a 3D shape and aggregates these contributions
- Bidirectional Reflectance Distribution Functions (BRDFs)
  - Cook and Torrance (specular reflection)
  - Lommel-Seeliger (diffuse reflection)

## ■ Post-processing of the posterior PDF

- Cluster analysis to identify the modes of the resulting posterior PDF
- Clustering algorithm: Density-Based Spatial Clustering of Applications with Noise (DBSCAN)
- Analysis of PDF modes to identify the most probable attitude

# Results

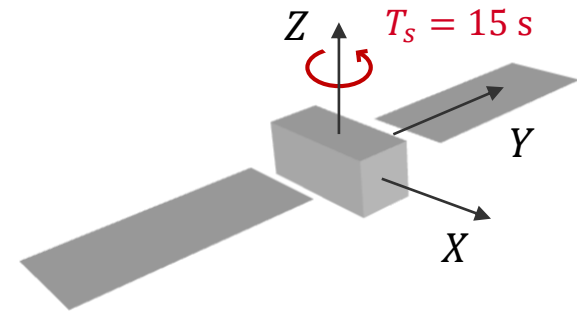
# Previous considerations

## ■ General considerations

- Estimate the initial attitude of an RSO from simulated observations
- Assume that the RSO's shape, size and surface optical properties, as well as the AOD, are known
- Assume that the uncontrolled RSO follows a spinning attitude law over the tracking duration
- Spinning attitude law parameters: RSO's orientation, spin rotation axis and spin angular velocity
- Additional assumptions:
  - Spin angular velocity can be estimated using a period-finding method (e.g. Lomb-Scargle periodogram)
  - In the long-term, the RSO eventually rotates about its principal axis of maximum inertia (flat spin)

## ■ RSO model properties

- Low Earth Orbit (LEO) at an altitude of 2000 km
- Satellite's platform optical properties:  $K_d = 0.15$ ,  $K_s = 0.7$
- Solar array optical properties:  $K_d = 0$ ,  $K_s = 0.1$



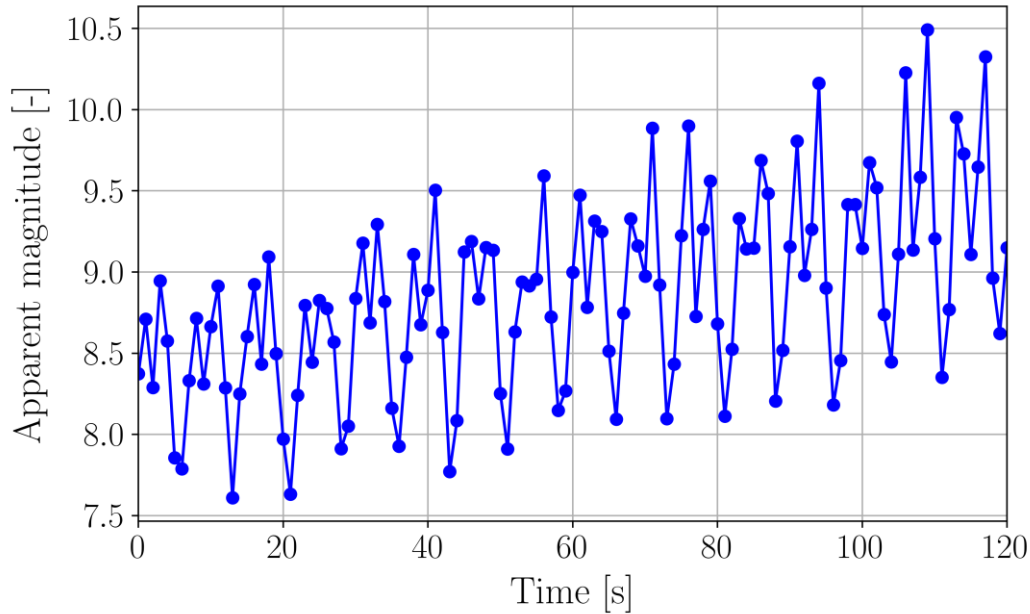


# Results

- **Case 1: unimodal posterior PDF**

# Results. Case 1: unimodal posterior PDF

## ■ Case 1: light curve



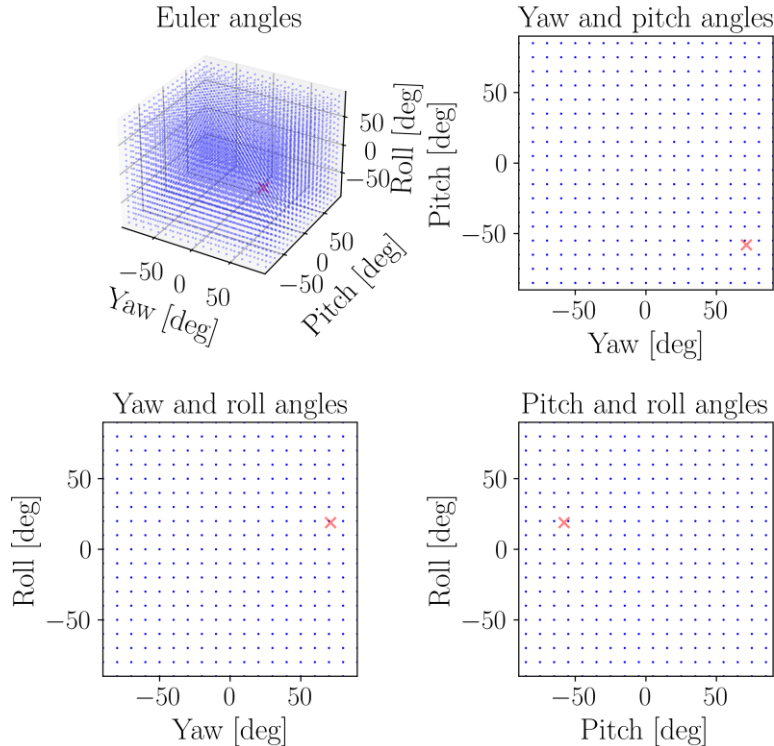
Attitude:  $\mathbf{x}_0 = [71, -58, 19]$  deg

Sensor noise:  $\sigma = 0.1$

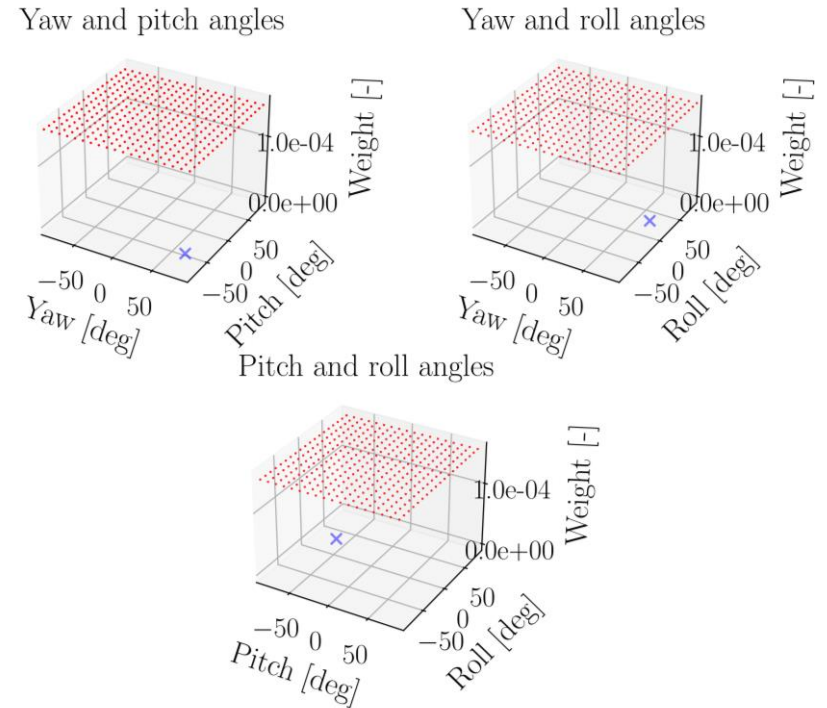
# Results. Case 1: unimodal posterior PDF

## ■ Case 1: particle filter execution

Initial uniform distribution



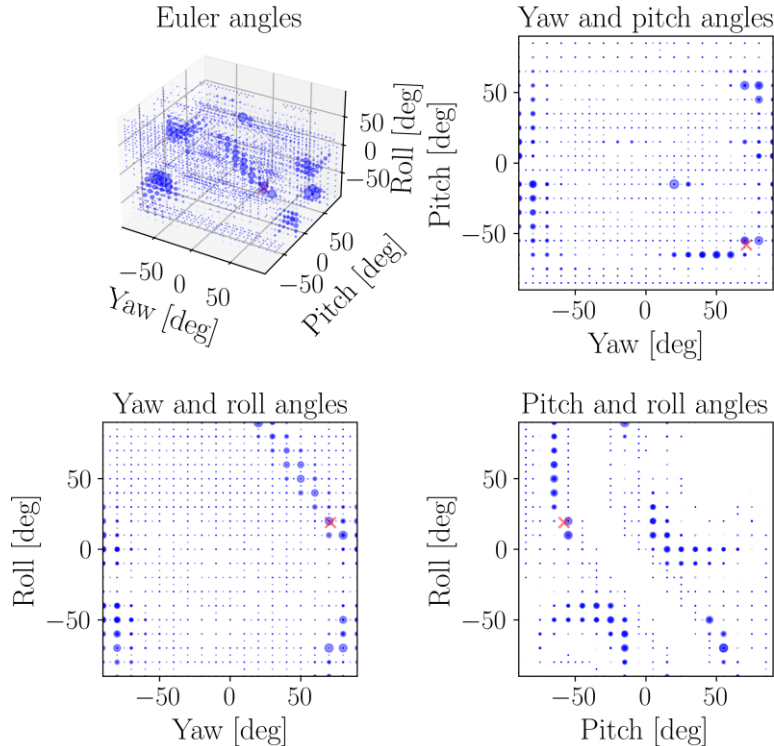
Initial uniform distribution



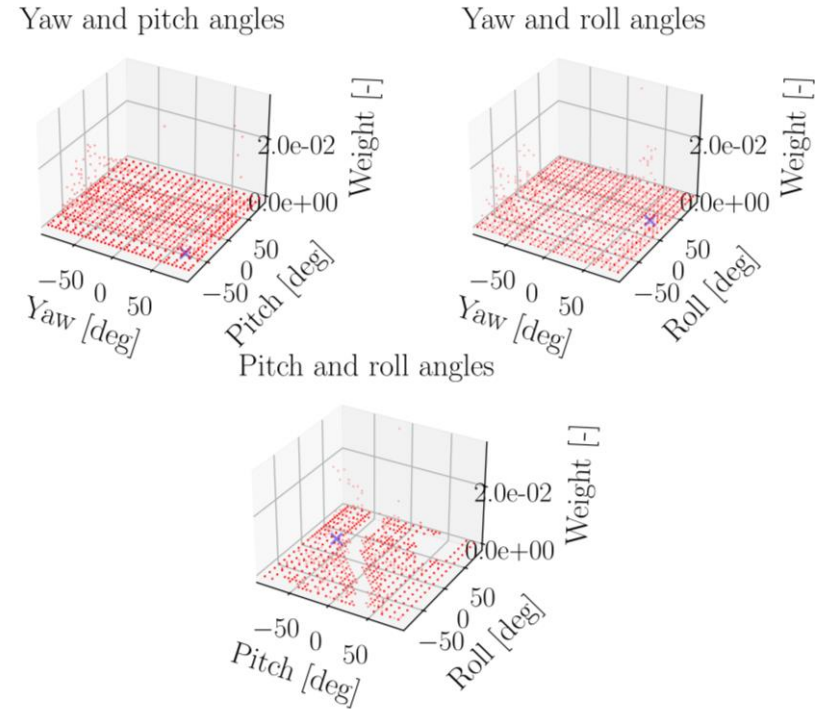
# Results. Case 1: unimodal posterior PDF

## ■ Case 1: particle filter execution

PF iteration number = 1



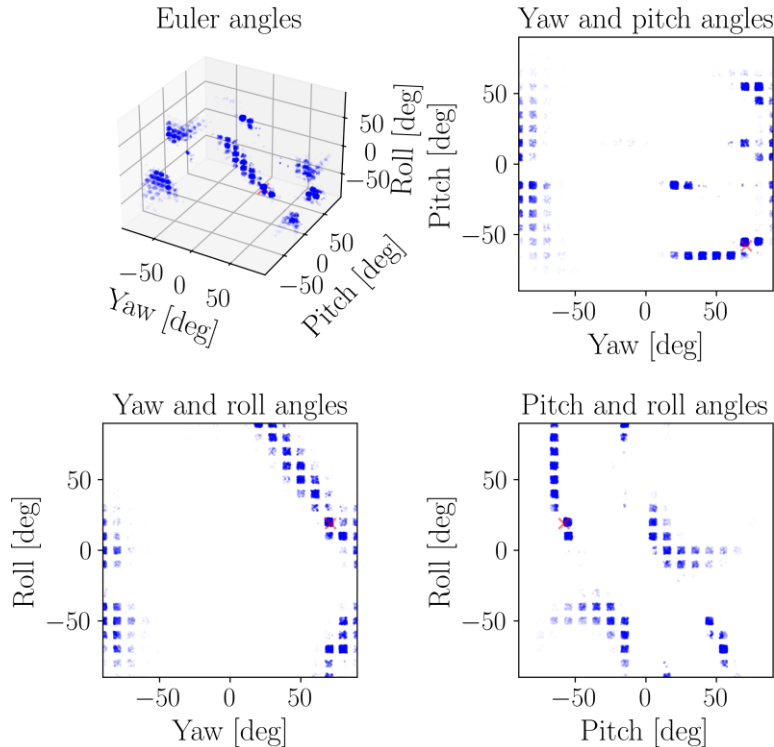
PF iteration number = 1



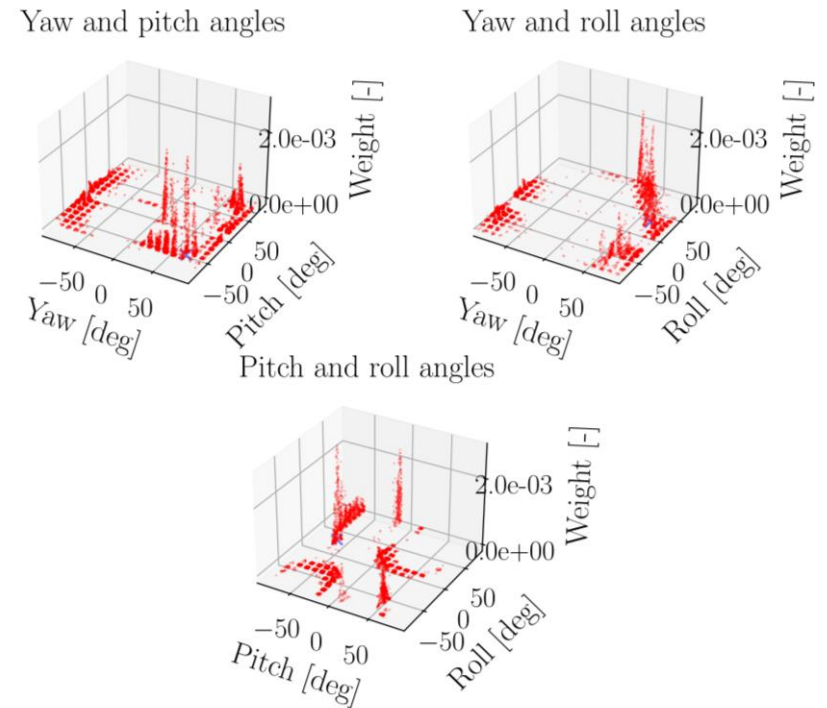
# Results. Case 1: unimodal posterior PDF

## ■ Case 1: particle filter execution

PF iteration number = 2



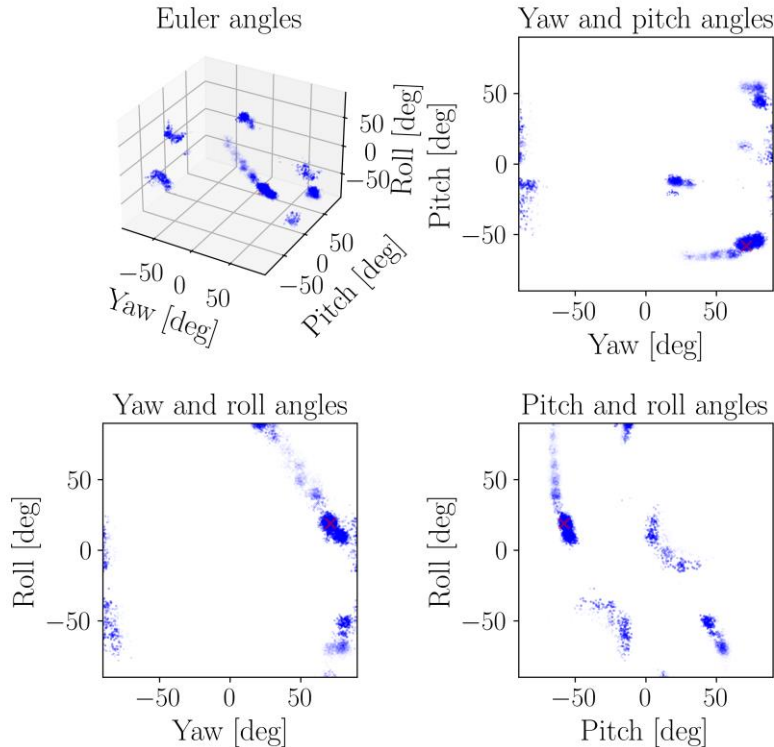
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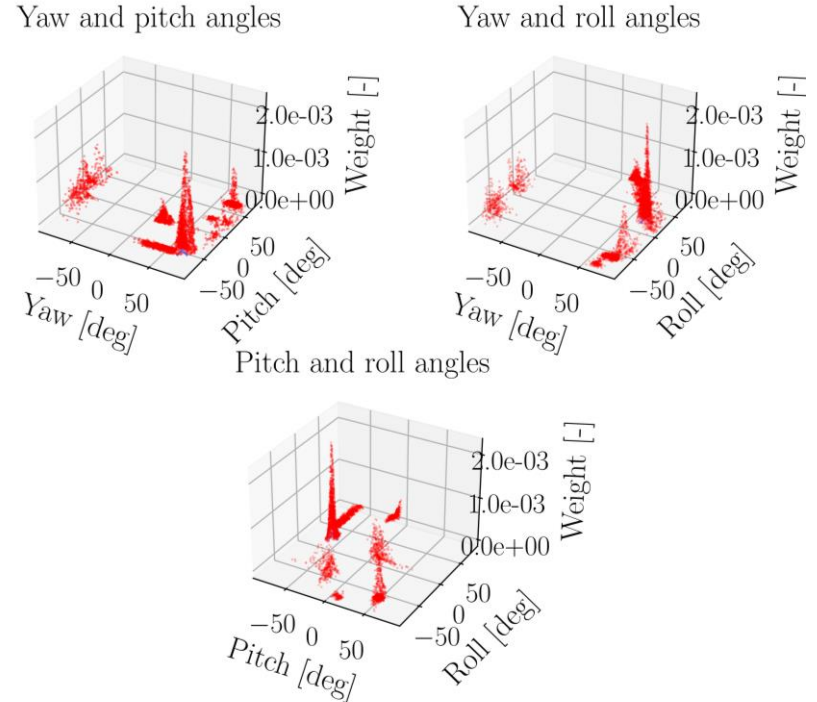
# Results. Case 1: unimodal posterior PDF

## ■ Case 1: particle filter execution

PF iteration number = 5



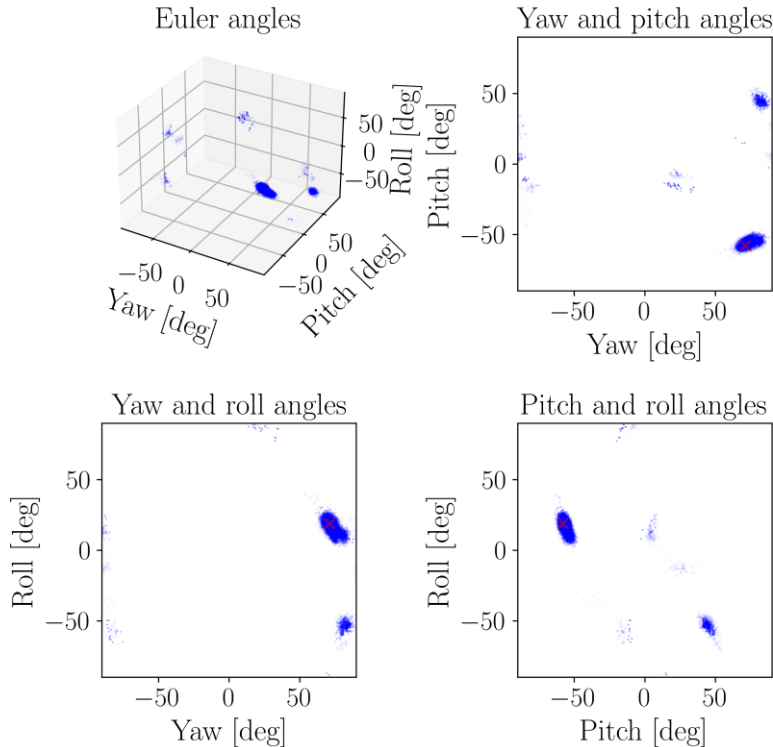
PF iteration number = 5



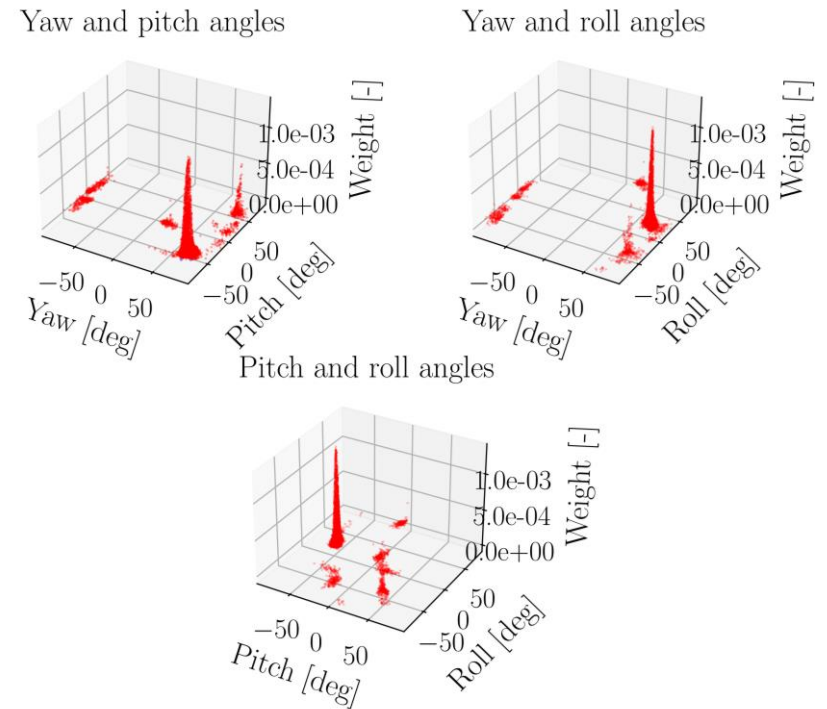
# Results. Case 1: unimodal posterior PDF

## ■ Case 1: particle filter execution

PF iteration number = 10



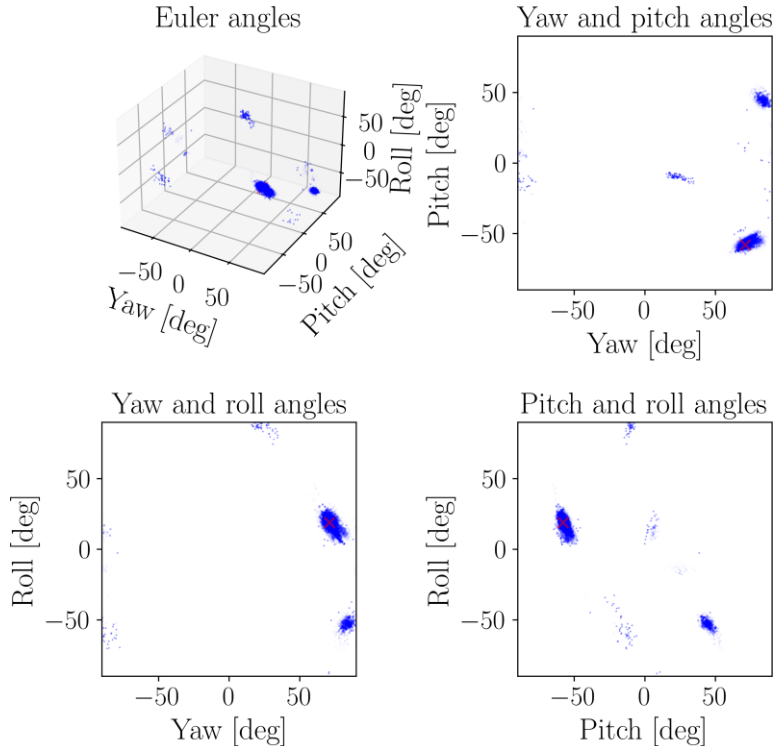
PF iteration number = 10



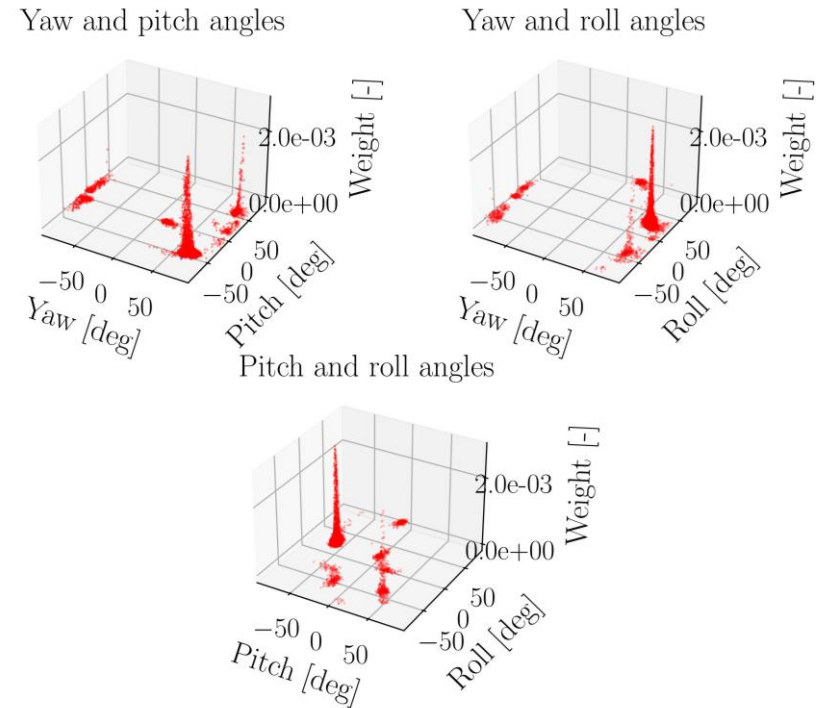
# Results. Case 1: unimodal posterior PDF

## ■ Case 1: particle filter execution

PF iteration number = 15



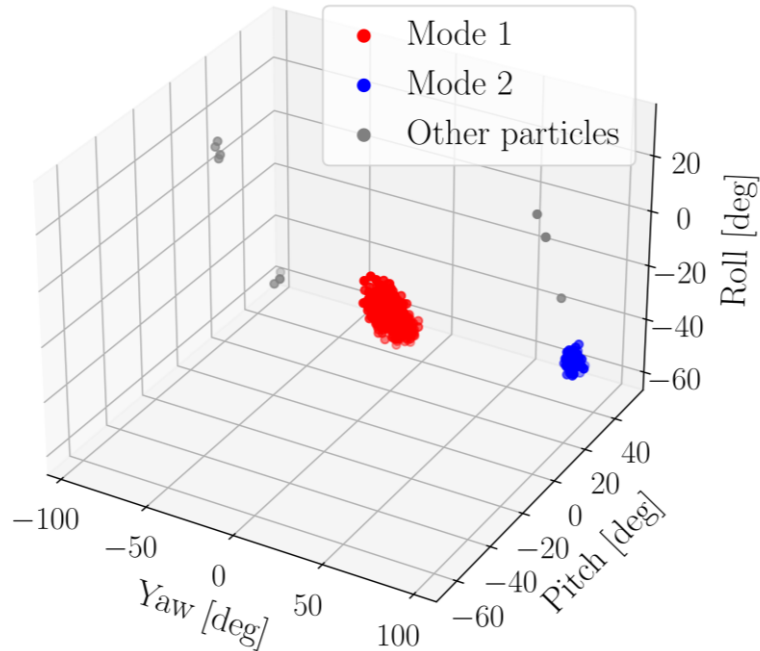
PF iteration number = 15





# Results. Case 1: unimodal posterior PDF

## ■ Case 1: cluster and posterior PDF analyses



True attitude:  $\mathbf{x}_0 = [71, -58, 19]$  deg

### Mode 1

$$W = 0.92 \quad W_N = 1.5 \times 10^{-4}$$

$$\mathbf{x}_0 = [71.75, -57.54, 18.74] \text{ deg}$$

$$\boldsymbol{\sigma} = [0.03, 0.02, 0.04] \text{ deg}$$

### Mode 2

$$W = 0.08 \quad W_N = 1.5 \times 10^{-4}$$

$$\mathbf{x}_0 = [84.43, 44.25, 52.91] \text{ deg}$$

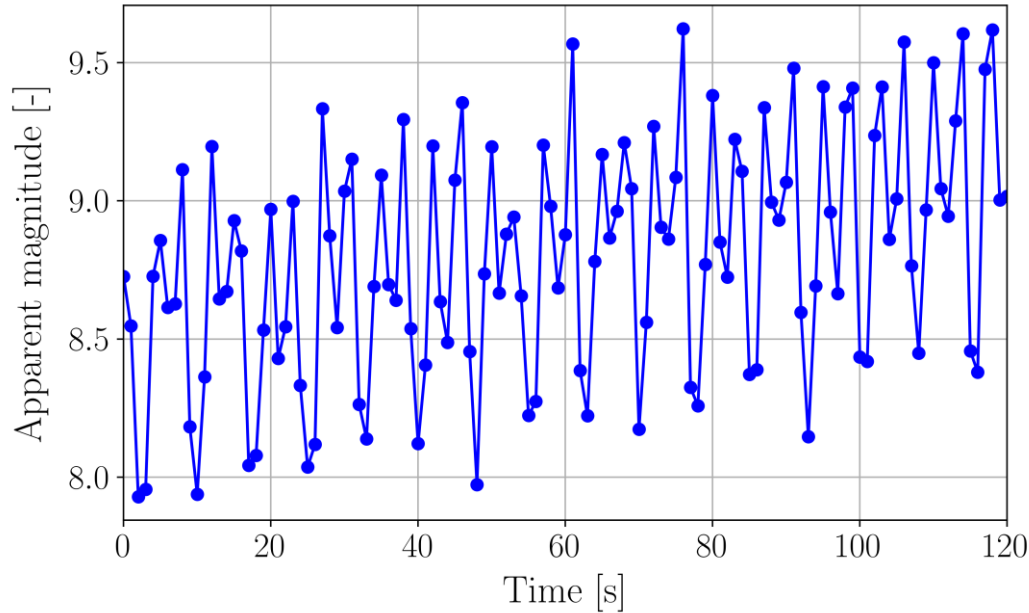
$$\boldsymbol{\sigma} = [0.09, 0.09, 0.10] \text{ deg}$$

# Results

- **Case 2: multimodal posterior PDF**

# Results. Case 2: multimodal posterior PDF

## ■ Case 2A: light curve



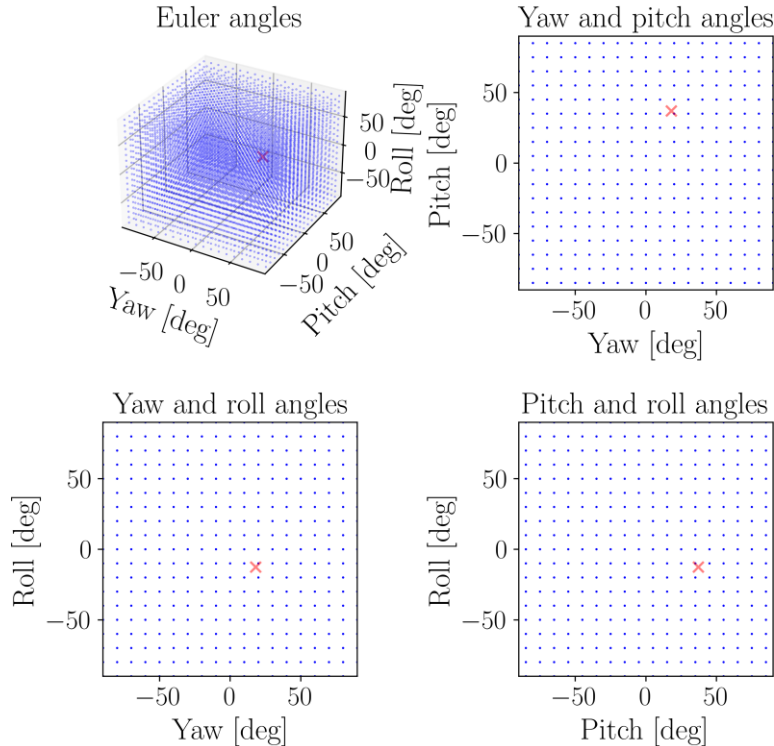
Attitude:  $\mathbf{x}_0 = [18, 37, -12.5]$  deg

Sensor noise:  $\sigma = 0.1$

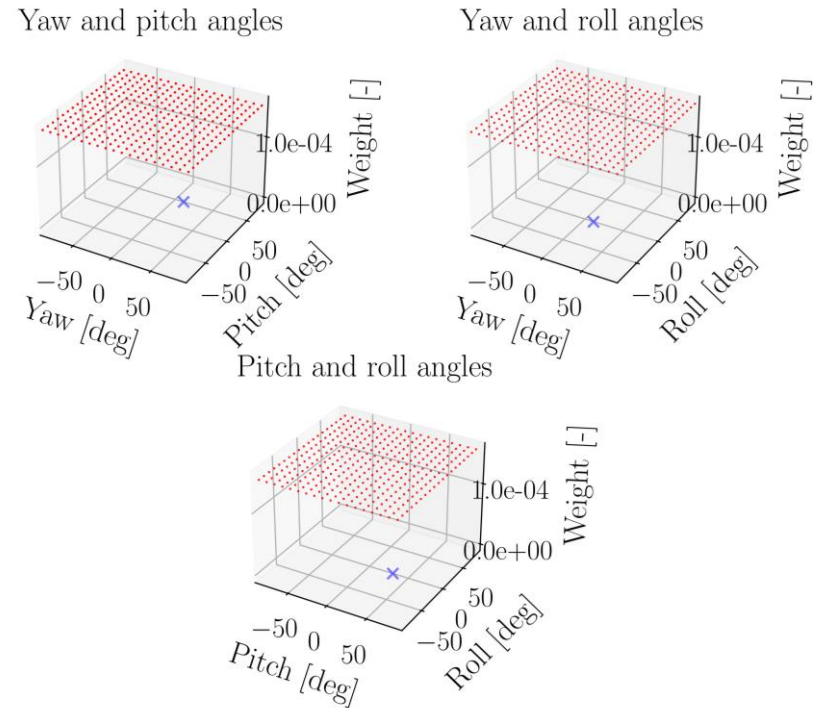
# Results. Case 2: multimodal posterior PDF

## ■ Case 2A: particle filter execution

Initial uniform distribution



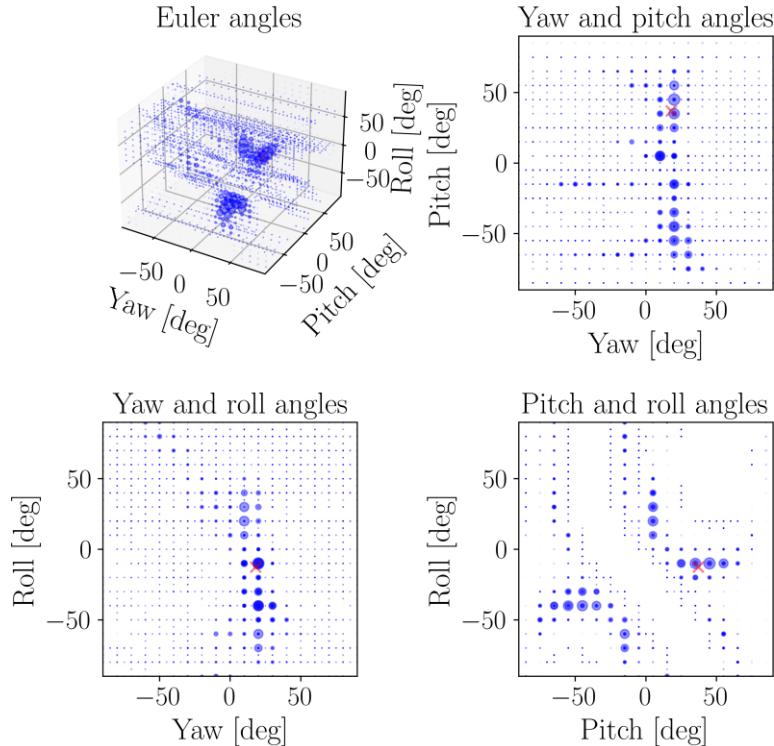
Initial uniform distribution



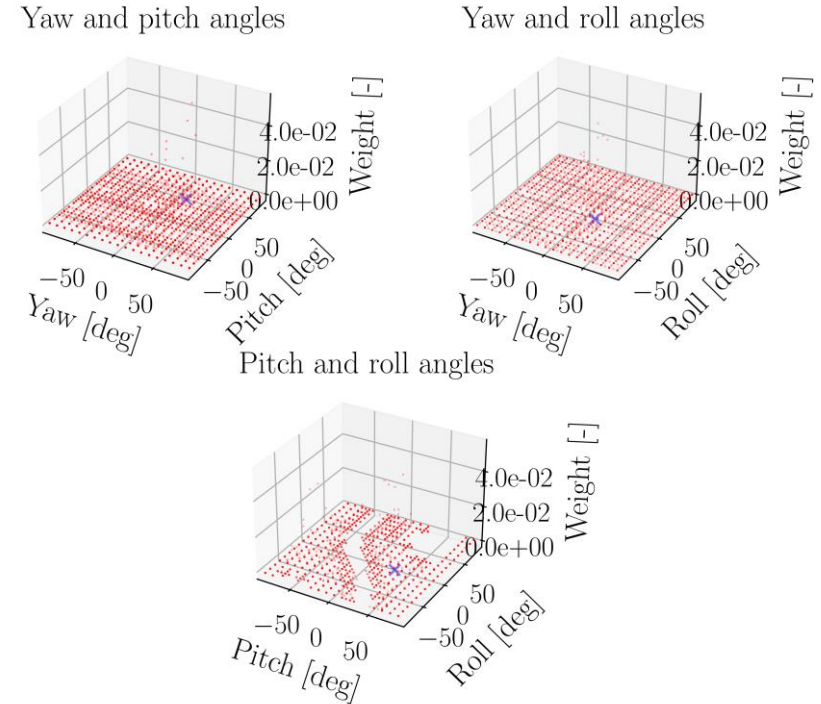
# Results. Case 2: multimodal posterior PDF

## ■ Case 2A: particle filter execution

PF iteration number = 1



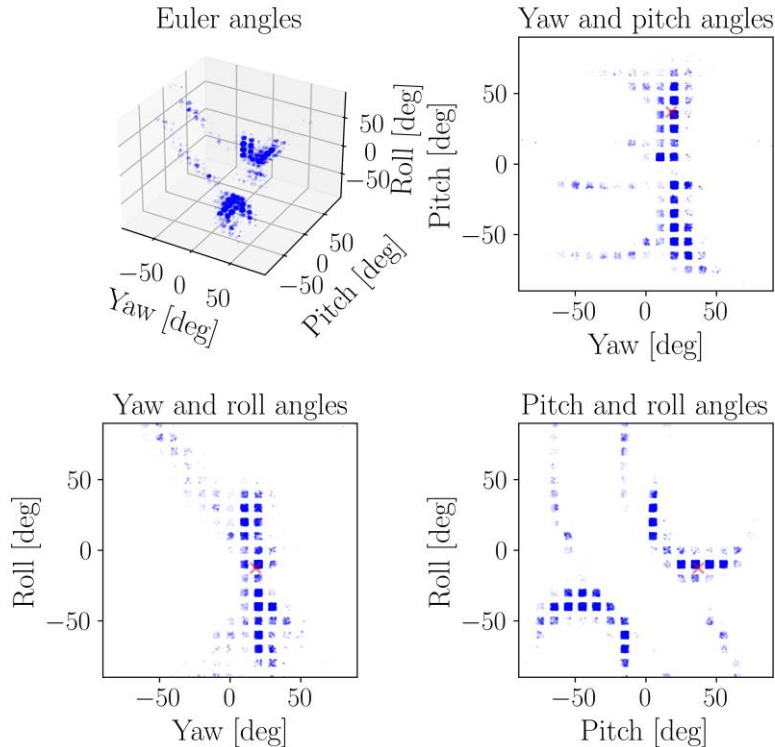
PF iteration number = 1



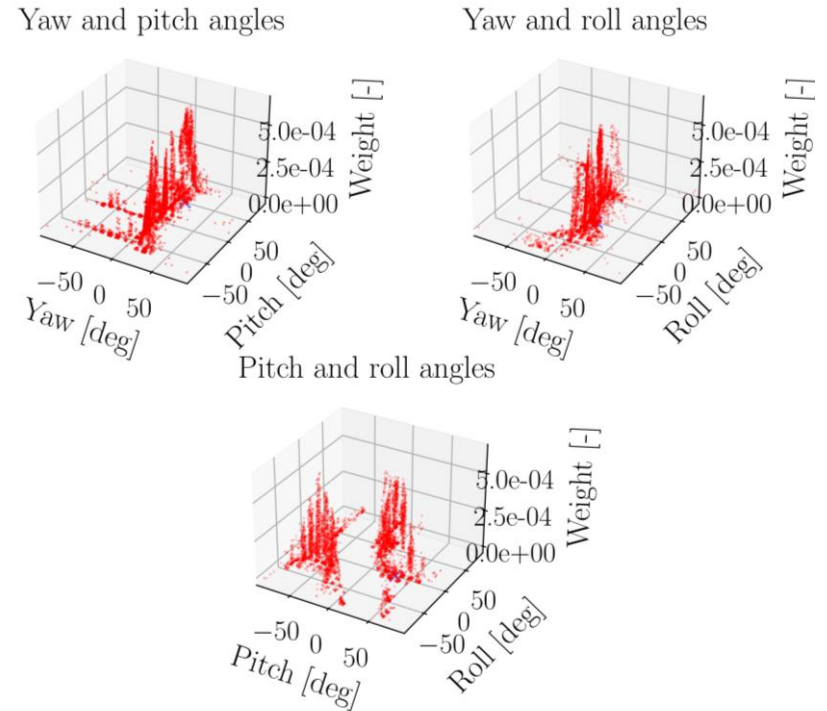
# Results. Case 2: multimodal posterior PDF

## ■ Case 2A: particle filter execution

PF iteration number = 2



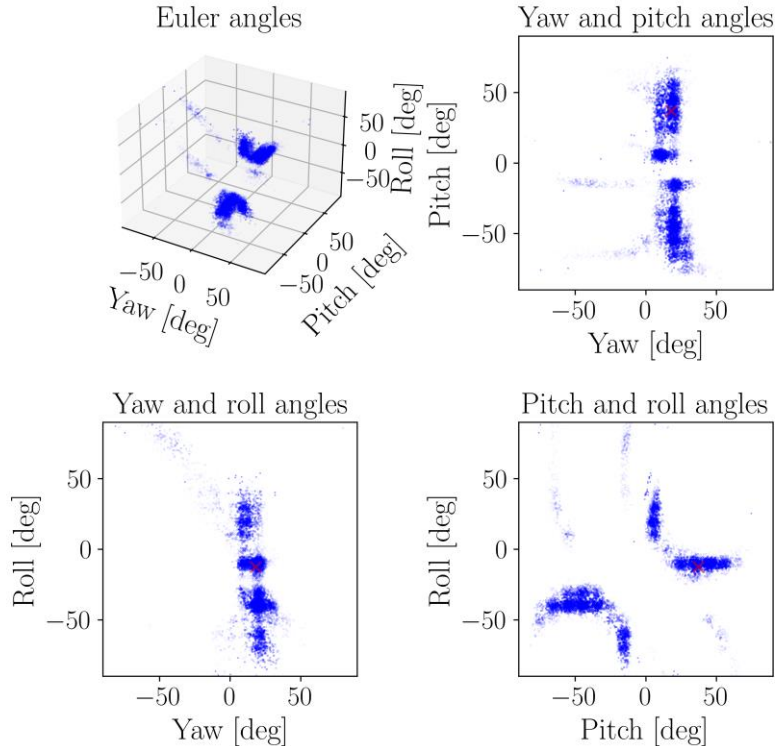
PF iteration number = 2



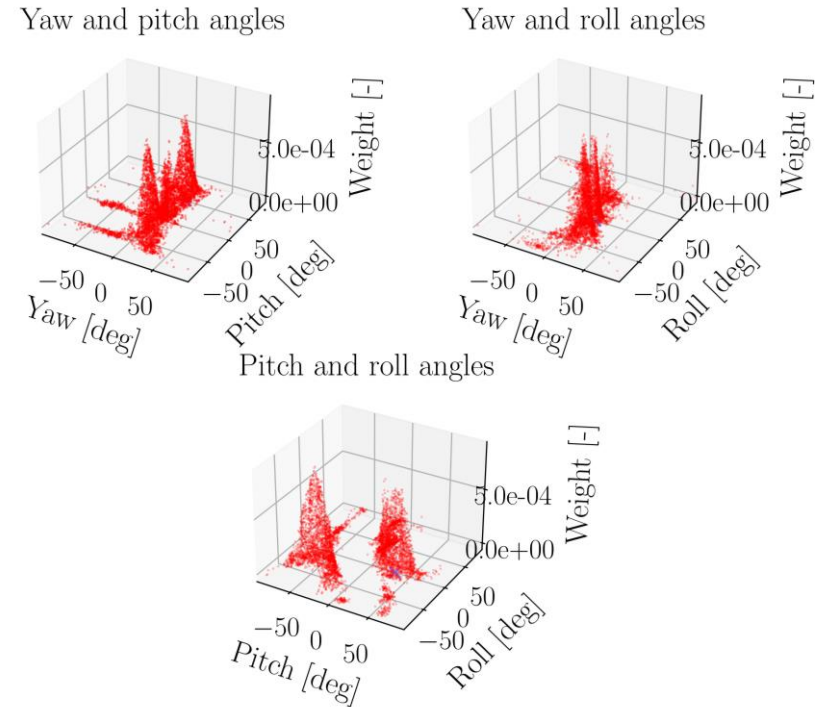
# Results. Case 2: multimodal posterior PDF

## ■ Case 2A: particle filter execution

PF iteration number = 5



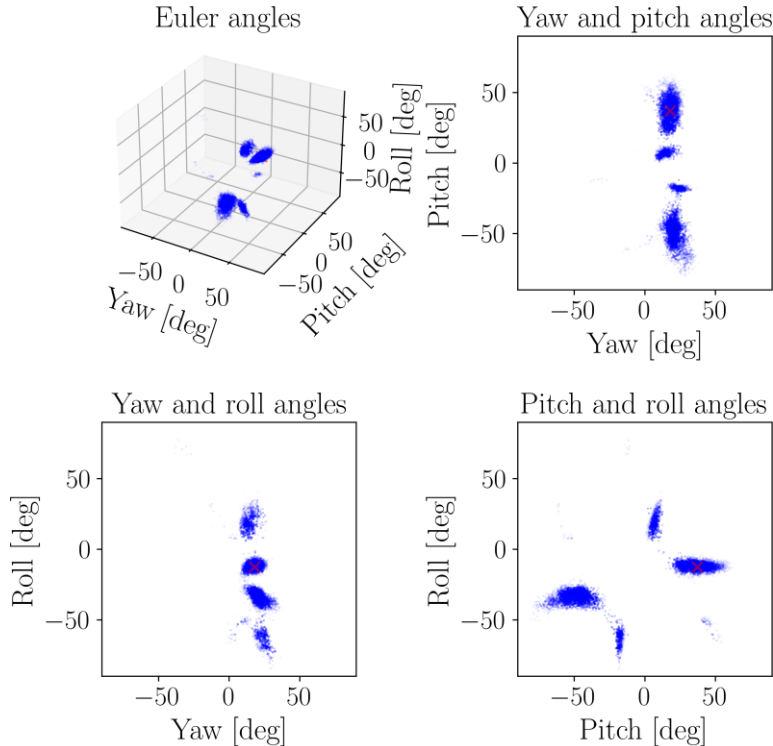
PF iteration number = 5



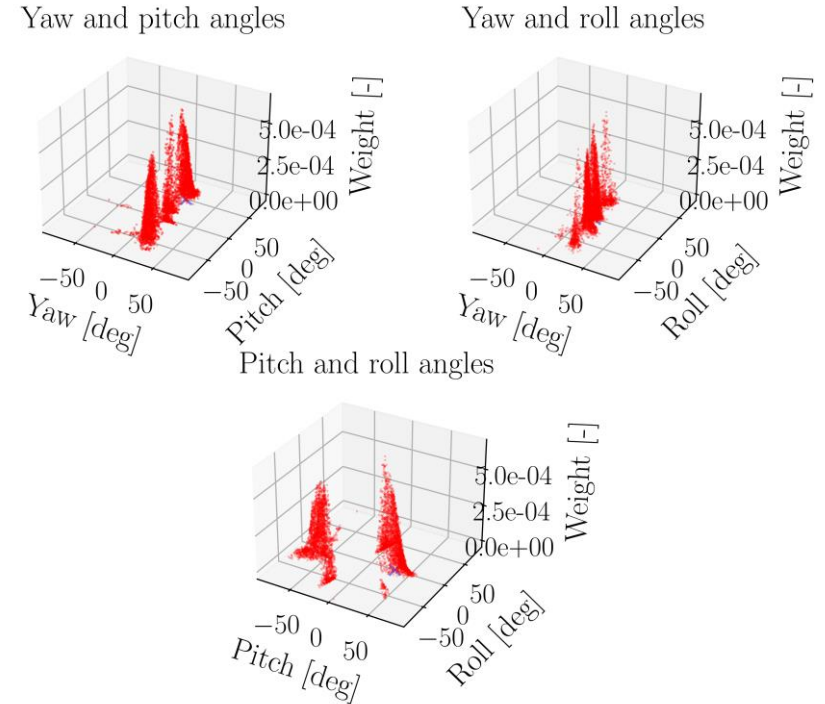
# Results. Case 2: multimodal posterior PDF

## ■ Case 2A: particle filter execution

PF iteration number = 10



PF iteration number = 10

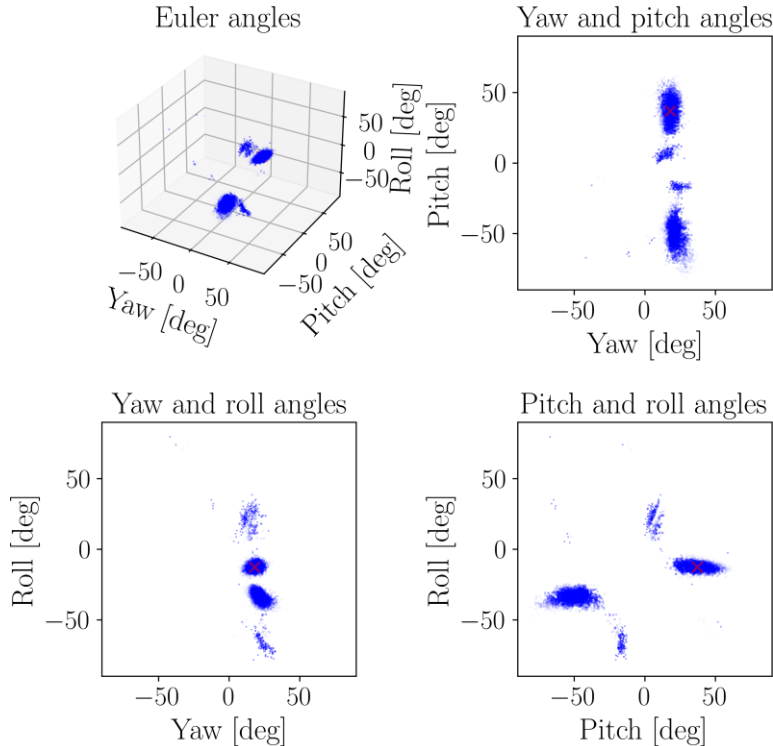




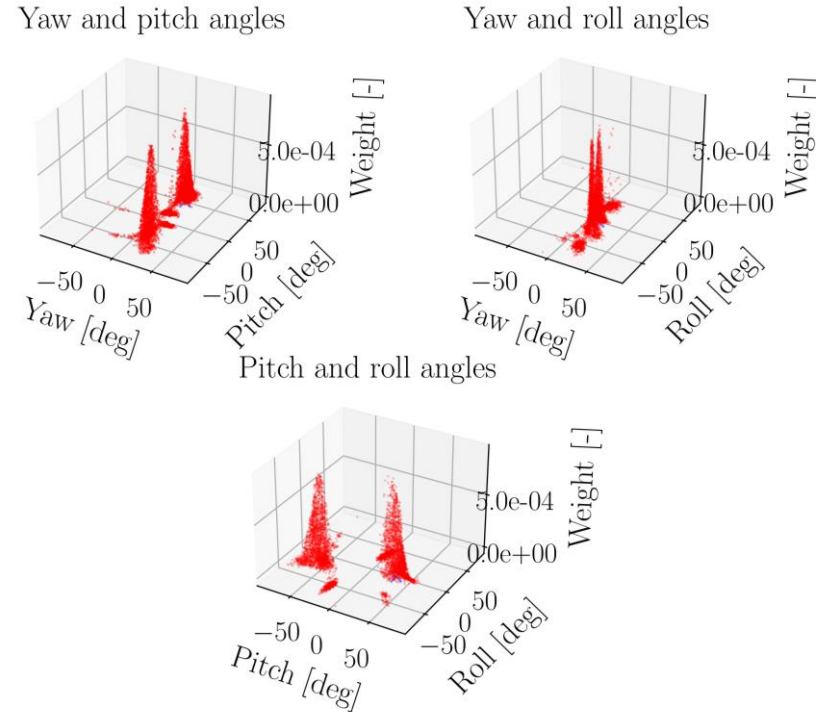
# Results. Case 2: multimodal posterior PDF

## ■ Case 2A: particle filter execution

PF iteration number = 15

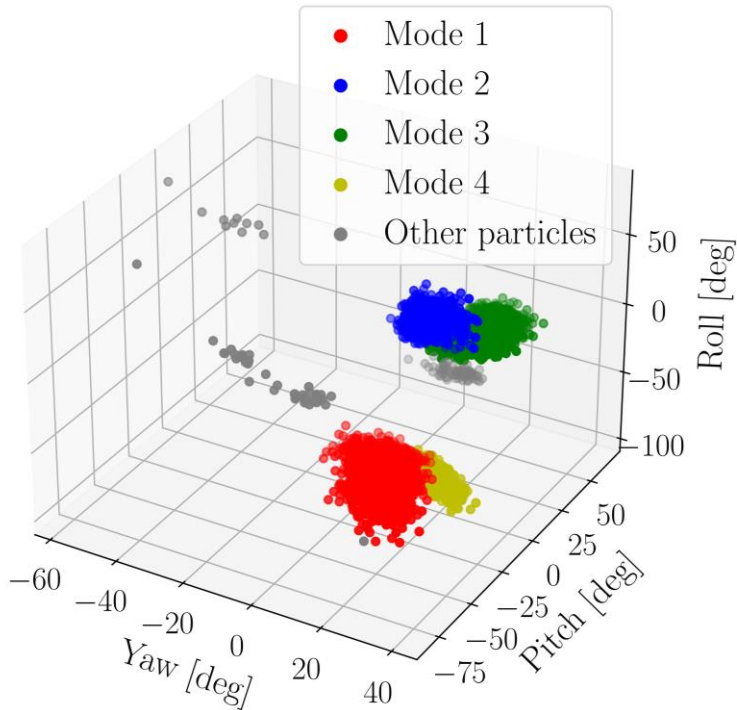


PF iteration number = 15



# Results. Case 2: multimodal posterior PDF

## ■ Case 2A: cluster and posterior PDF analyses



True attitude:  $\mathbf{x}_0 = [18, 37, -12.5]$  deg

### Mode 1

$$W = 0.52 \quad W_N = 2.1 \times 10^{-4}$$

$$\mathbf{x}_0 = [21.58, -49.85, -33.42] \text{ deg}$$

$$\boldsymbol{\sigma} = [0.05, 0.13, 0.06] \text{ deg}$$

### Mode 2

$$W = 10^{-4} \quad W_N = 1.2 \times 10^{-5}$$

$$\mathbf{x}_0 = [18.02, 9.03, 14.78] \text{ deg}$$

$$\boldsymbol{\sigma} = [0.09, 0.07, 0.17] \text{ deg}$$

### Mode 3

$$W = 0.47 \quad W_N = 1.9 \times 10^{-4}$$

$$\mathbf{x}_0 = [18.38, 36.41, -12.74] \text{ deg}$$

$$\boldsymbol{\sigma} = [0.05, 0.13, 0.04] \text{ deg}$$

### Mode 4

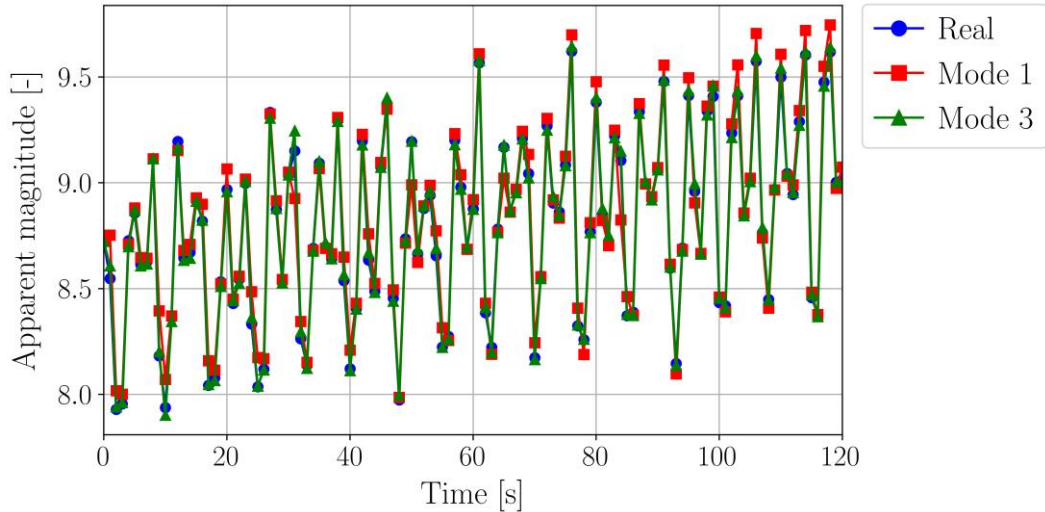
$$W = 7 \times 10^{-4} \quad W_N = 1.2 \times 10^{-6}$$

$$\mathbf{x}_0 = [25.44, -20.91, -56.28] \text{ deg}$$

$$\boldsymbol{\sigma} = [0.05, 0.04, 0.07] \text{ deg}$$

# Results. Case 2: multimodal posterior PDF

## ■ Case 2A: simulated light curves for the relevant modes



Possible solution: stereoscopic measurements

True attitude:  $\mathbf{x}_0 = [18, 37, -12.5]$  deg

### Mode 1

$$W = 0.52 \quad W_N = 2.1 \times 10^{-4}$$

$$\mathbf{x}_0 = [21.58, -49.85, -33.42] \text{ deg}$$

$$\boldsymbol{\sigma} = [0.05, 0.13, 0.06] \text{ deg}$$

### Mode 3

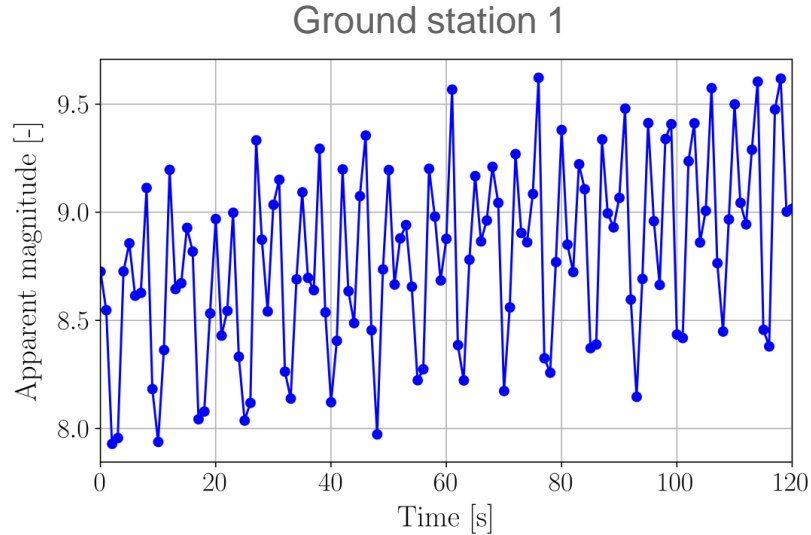
$$W = 0.47 \quad W_N = 1.9 \times 10^{-4}$$

$$\mathbf{x}_0 = [18.38, 36.41, -12.74] \text{ deg}$$

$$\boldsymbol{\sigma} = [0.05, 0.13, 0.04] \text{ deg}$$

# Results. Case 2: multimodal posterior PDF

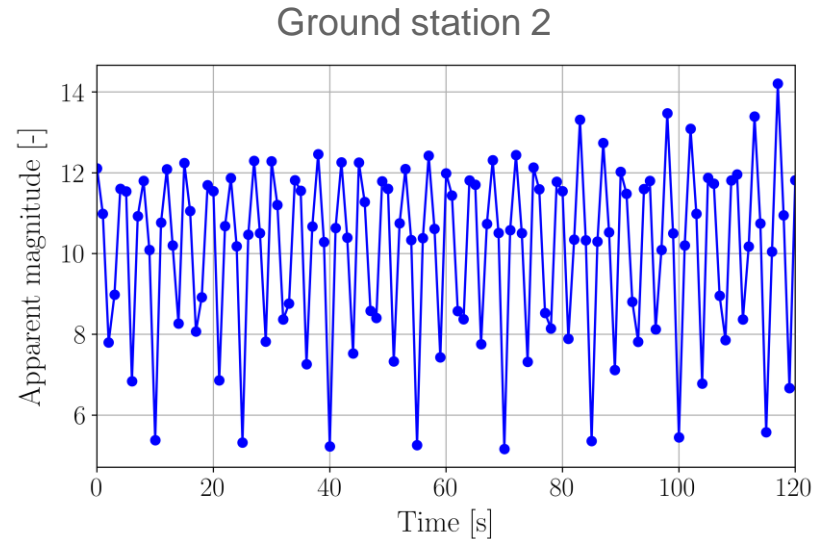
- Case 2B: light curves from 2 ground stations during the same time interval



Attitude:  $\mathbf{x}_0 = [18, 37, -12.5]$  deg

Sensors noise:  $\sigma = 0.1$

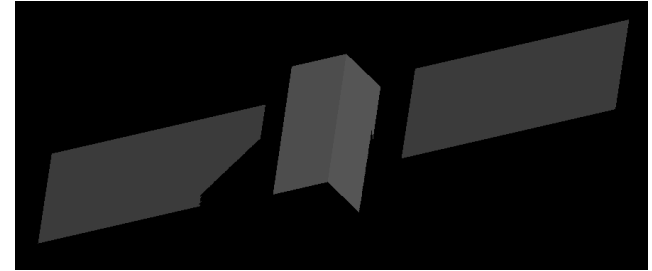
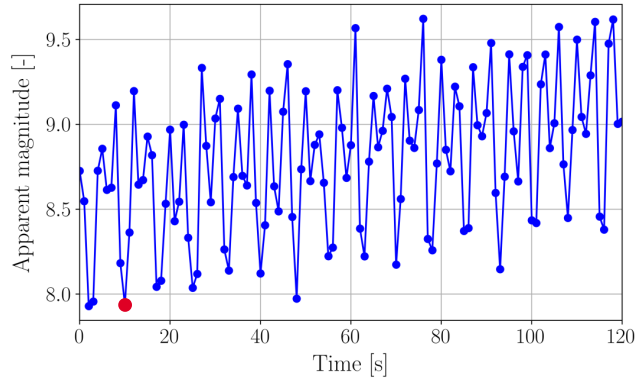
GS 2: 30° N, 15° E w.r.t. GS 1



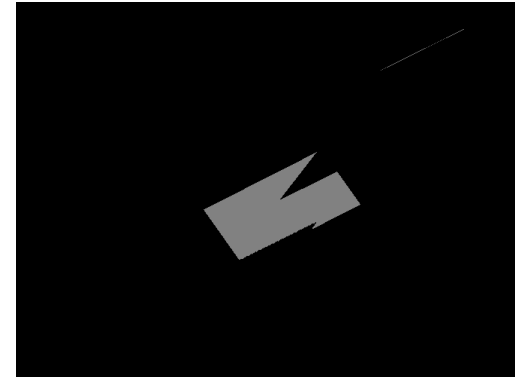
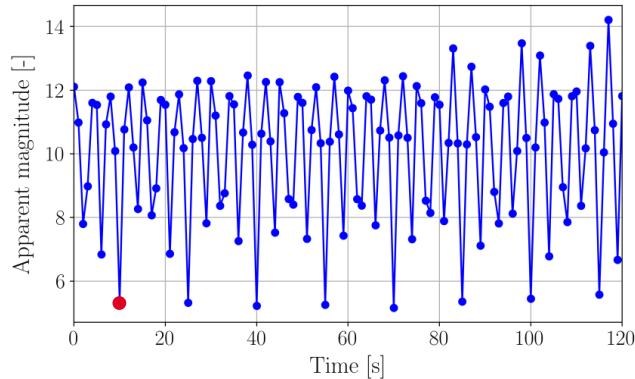
# Results. Case 2: multimodal posterior PDF

- Case 2B: light curves from 2 ground stations during the same time interval

Ground station 1



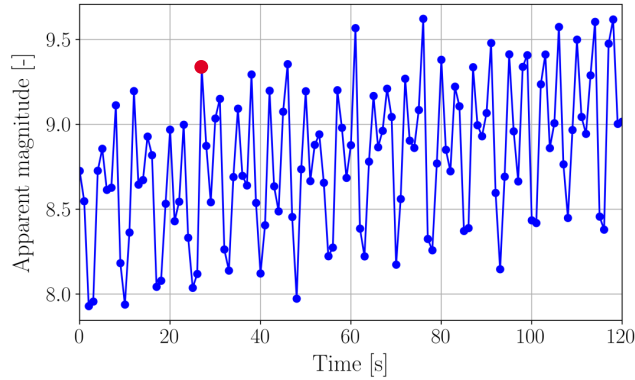
Ground station 2



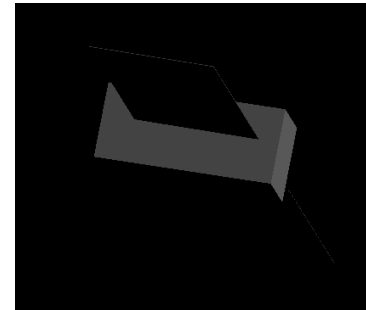
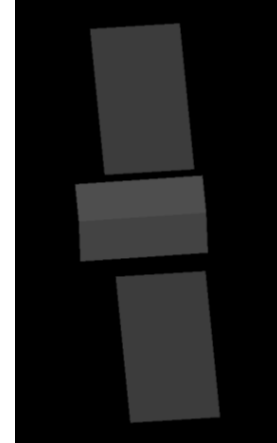
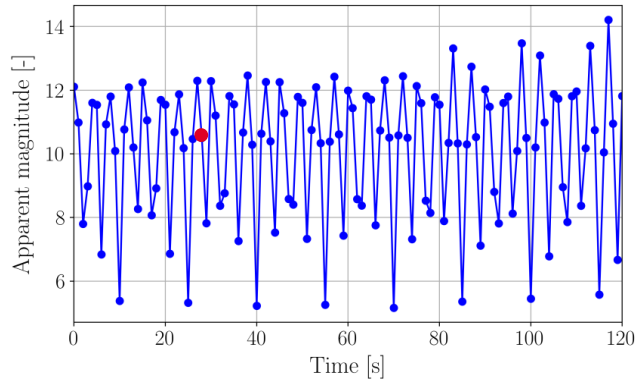
# Results. Case 2: multimodal posterior PDF

- Case 2B: light curves from 2 ground stations during the same time interval

Ground station 1



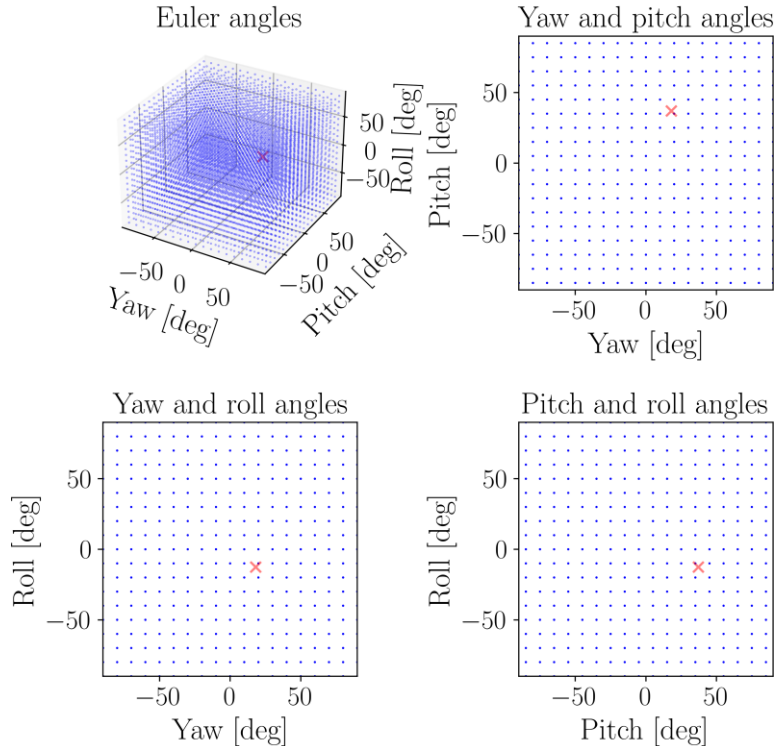
Ground station 2



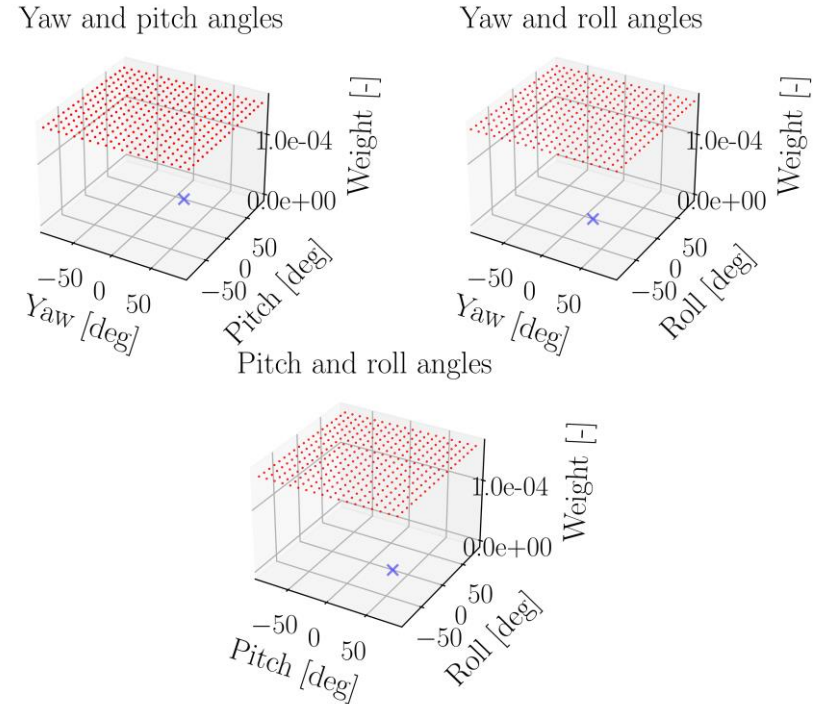
# Results. Case 2: multimodal posterior PDF

## ■ Case 2B: particle filter execution

Initial uniform distribution



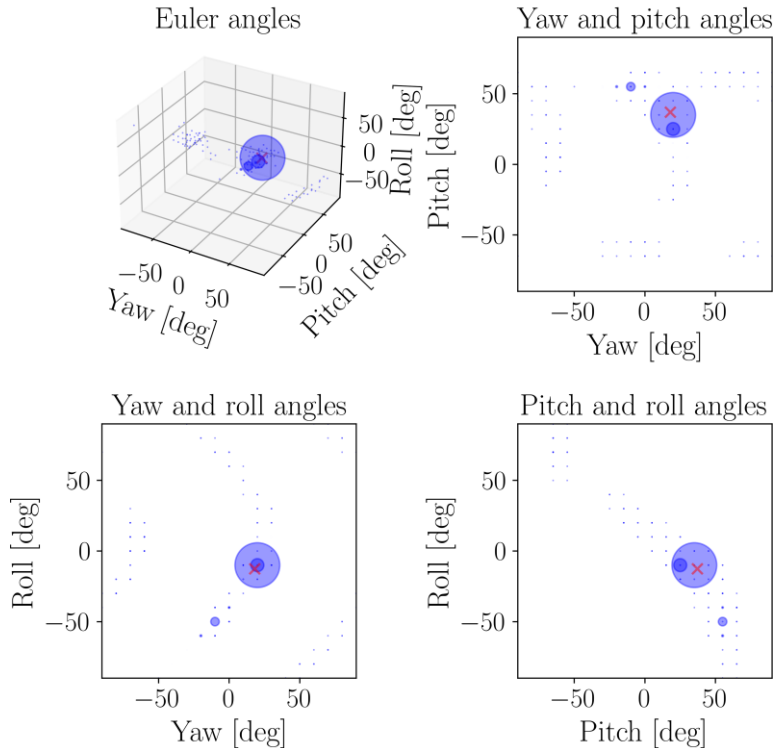
Initial uniform distribution



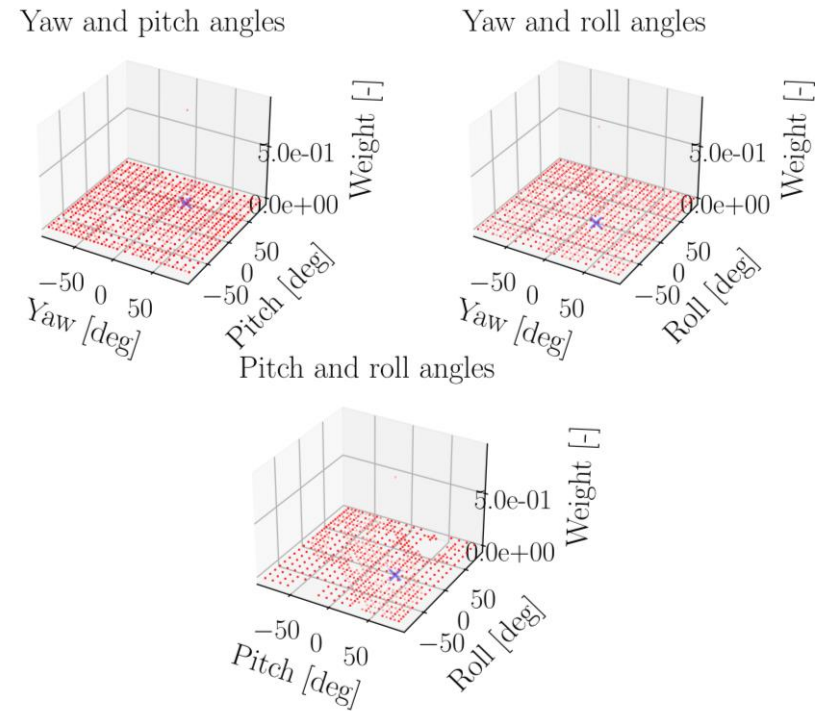
# Results. Case 2: multimodal posterior PDF

## ■ Case 2B: particle filter execution

PF iteration number = 1



PF iteration number = 1

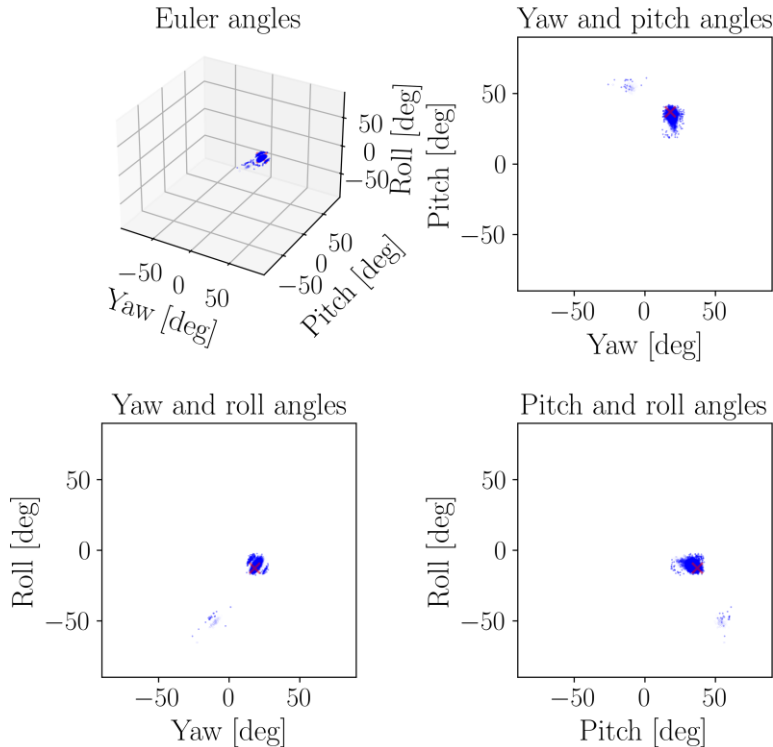




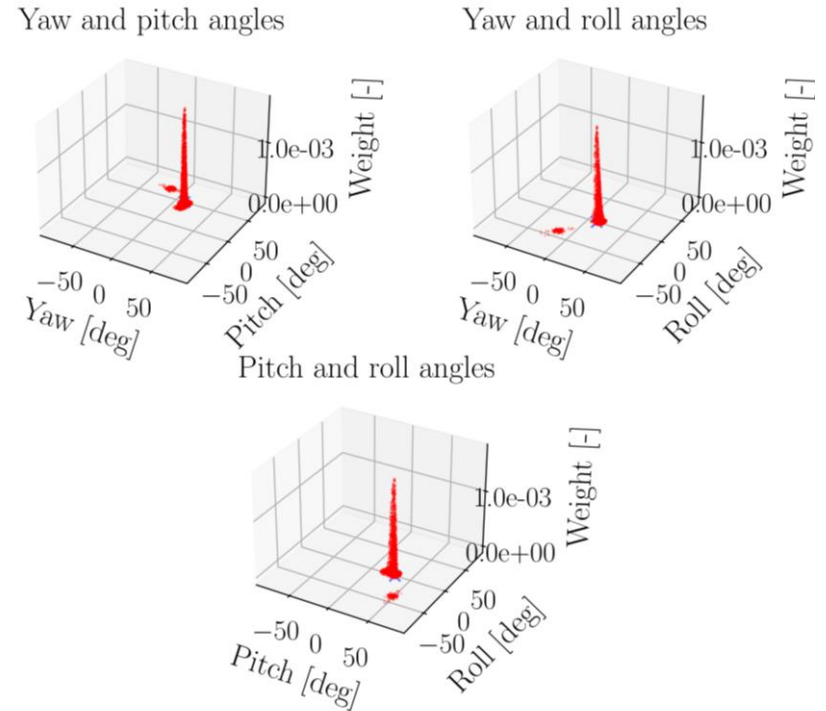
# Results. Case 2: multimodal posterior PDF

## ■ Case 2B: particle filter execution

PF iteration number = 5



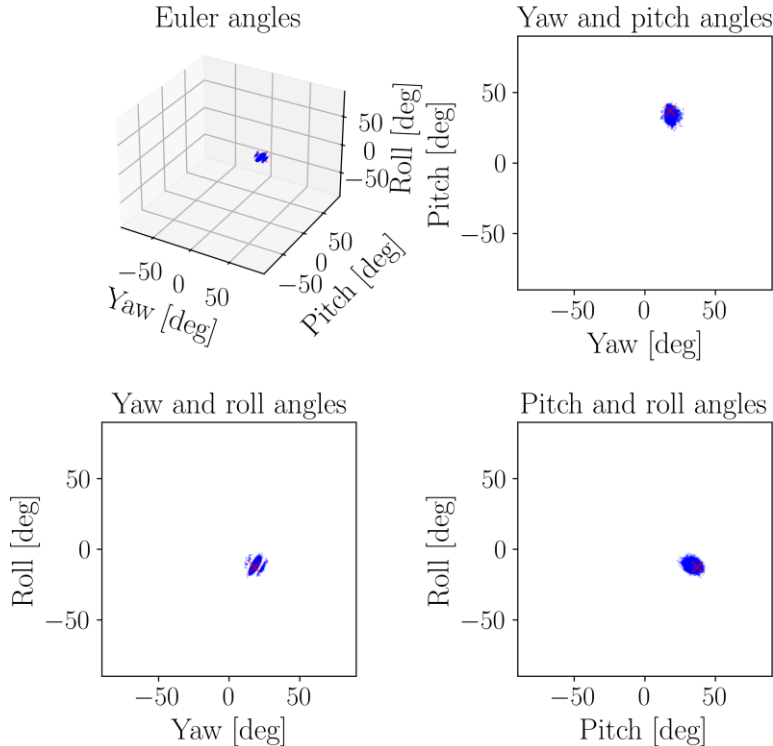
PF iteration number = 5



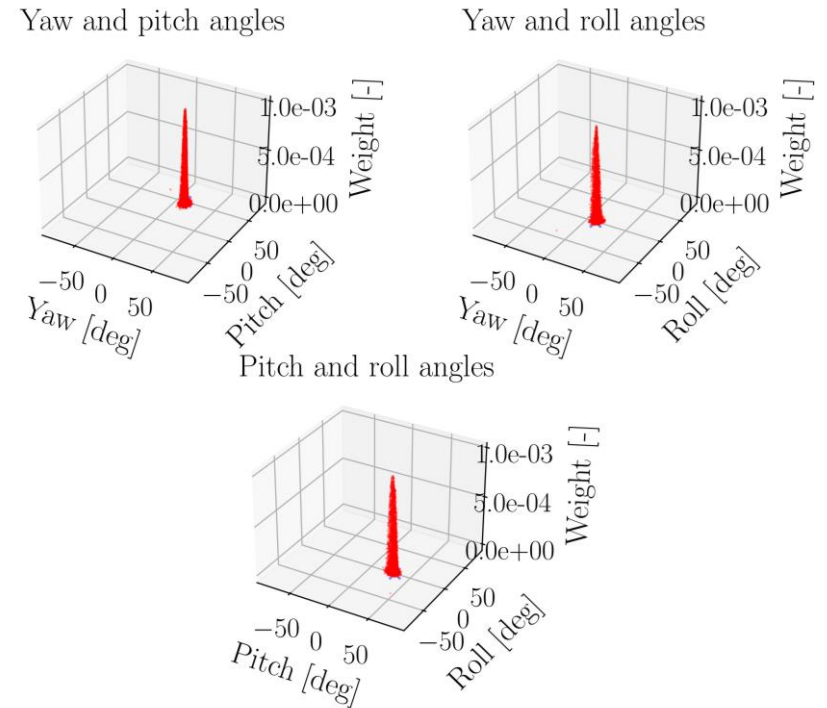
# Results. Case 2: multimodal posterior PDF

## ■ Case 2B: particle filter execution

PF iteration number = 10



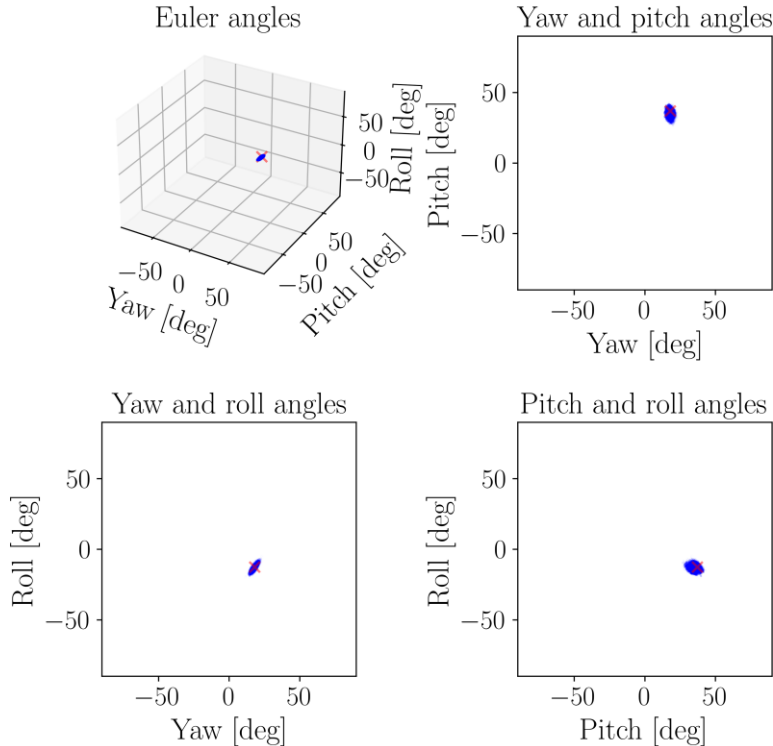
PF iteration number = 10



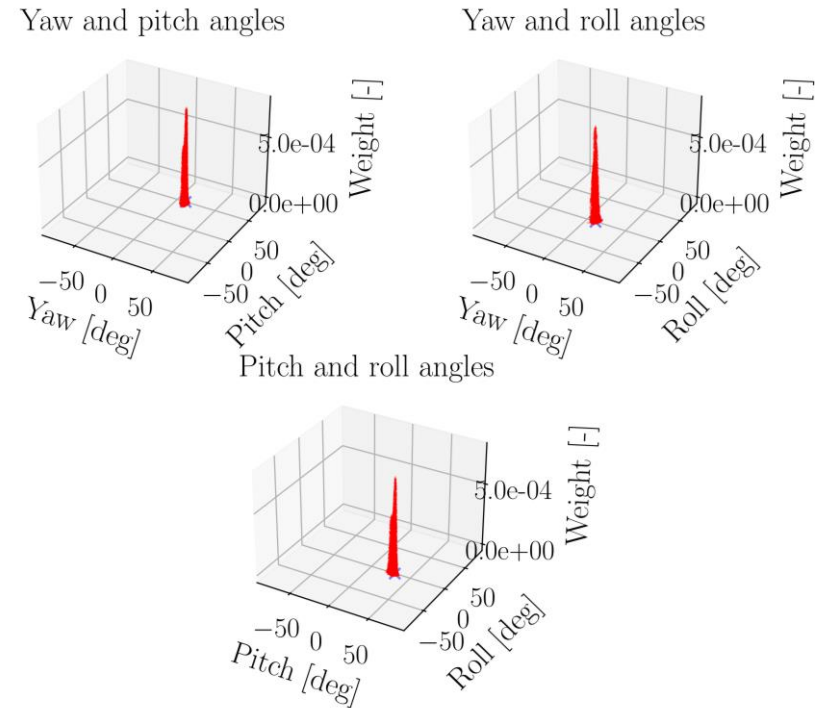
# Results. Case 2: multimodal posterior PDF

## ■ Case 2B: particle filter execution

PF iteration number = 15

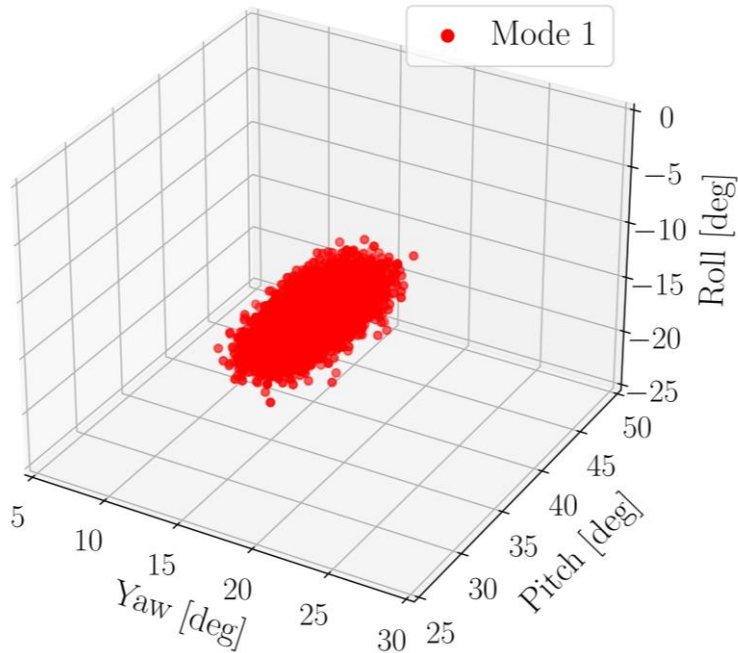


PF iteration number = 15



# Results. Case 2: multimodal posterior PDF

## ■ Case 2B: cluster and posterior PDF analyses



True attitude:  $\mathbf{x}_0 = [18, 37, -12.5]$  deg

Mode 1
$W = 1.0$
$\mathbf{x}_0 = [17.74, 35.66, -13.03]$ deg
$\boldsymbol{\sigma} = [0.01, 0.02, 0.02]$ deg

# Conclusions and future work

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## ■ Conclusions

- Understand the challenges of light curve inversion for attitude estimation
- Identify the limitations of sequential particle filters
- Develop the pseudo-batch particle filter with reinitialisation
- Analyse simulated test cases with single and multimodal posterior PDFs
- Use stereoscopic measurements to mitigate measurement ambiguities

## ■ Future work

- Improve the computational performance of the particle filter
- Relax assumptions regarding the RSO's attitude law and physical properties
- Extend analyses using real light curves

# Thank you

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