# Linearization of the Radiative Component of the Heat Transfer Equation for Space Thermal Analysis

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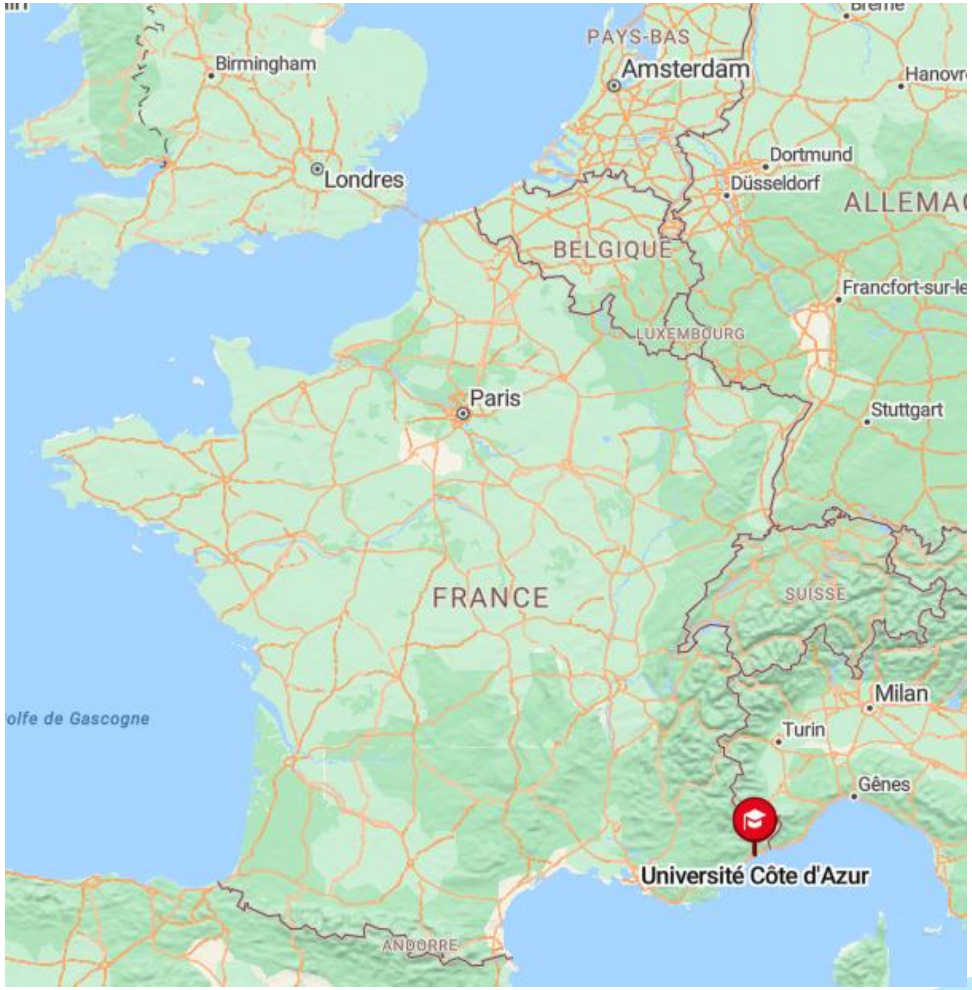
$8^{th}$ - $10^{th}$ October

# Côte d'Azur University

- Côte d’Azur University
- $1^{st}$ year of master in Mathematical Engineering (Scientific computing)
- Internship Dorea (SME)
- Partners :



# Topic

- **Real-time** space thermal simulation is a challenge (Digital Twin)

- Approach : **decrease computation time** dedicated to the resolution of the space thermal equation

- Proof Of Concept : 1st study for Dorea on the matter

# Problem statement

- Linearize the radiative component

$$MC_i\frac{dT_i}{dt} = Q_i + \sum_j GL_{ij}(T_j - T_i) + \sum_j \sigma GR_{ij}(T_j^4 - T_i^4)$$

- Compare with a traditionnal Runge-Kutta 4 method used as reference

$$y_{n+1} = y_n + h[\frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6}] \quad \text{with} \quad \begin{cases} k_1 = f(t_n, y_n) \\ k_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1) \\ k_3 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2) \\ k_4 = f(t_n + h, y_n + hk_3) \end{cases}$$

**Quantify the time gain relative to the error**

# State of the Art

- Linearization is subject to extensive research in the field of optimization [1]
- But still the log-linearization optimization dates back to 1961 [2]
- Idea : the **first-order Taylor** polynomial provides a **linear approximation**

$$f(x) = f(a) + f'(a)(x-a) + o(x-a) \quad [1]$$


[1]
M. Asghari, A. M. Fathollahi-Fard, S. M. J. Mirzapour Al-e hashem, and M. A. Dulebenets, "Transformation and linearization techniques in optimization: A state-of-the-art survey," *Mathematics*, vol. 10, no. 2, 2022.

[2]
R. E. Griffith and R. A. Stewart, "A nonlinear programming technique for the optimization of continuous processing systems," *Management Science*, vol. 7, no. 4, pp. 379–392, 1961.

# State of the Art

$$MC_i \frac{dT_i}{dt} = Q_i + \sum_j GL_{ij}(T_j - T_i) + \sum_j \sigma GR_{ij}(T_j^4 - T_i^4)$$

- How to linearize in thermal engineering ?
- (DSPE) Dutch Society for Precision Engineering [3]

Taylor expansion of $T_i \to {T_i}^4$ at $T_0$ to the 1st order :

$$T_i^4 = T_0^4 + 4T_0^3(T_i - T_0) + o(T_i - T_0)$$

- $T_0 = \frac{T_i + T_j}{2}$

$$MC_i \frac{dT_i}{dt} = Q_i + \sum_j GL_{ij}(T_j - T_i) + \sum_j \sigma GR_{ij} 4T_0^3(T_j - T_i)$$

[3] DSPE - Dutch Society for Precision Engineering, "Heat transfer: Radiation." https://www.dspe.nl/knowledge/thermomechanics/chapter-1-basics/1-2-heat-transfer/radiation/, 2024.

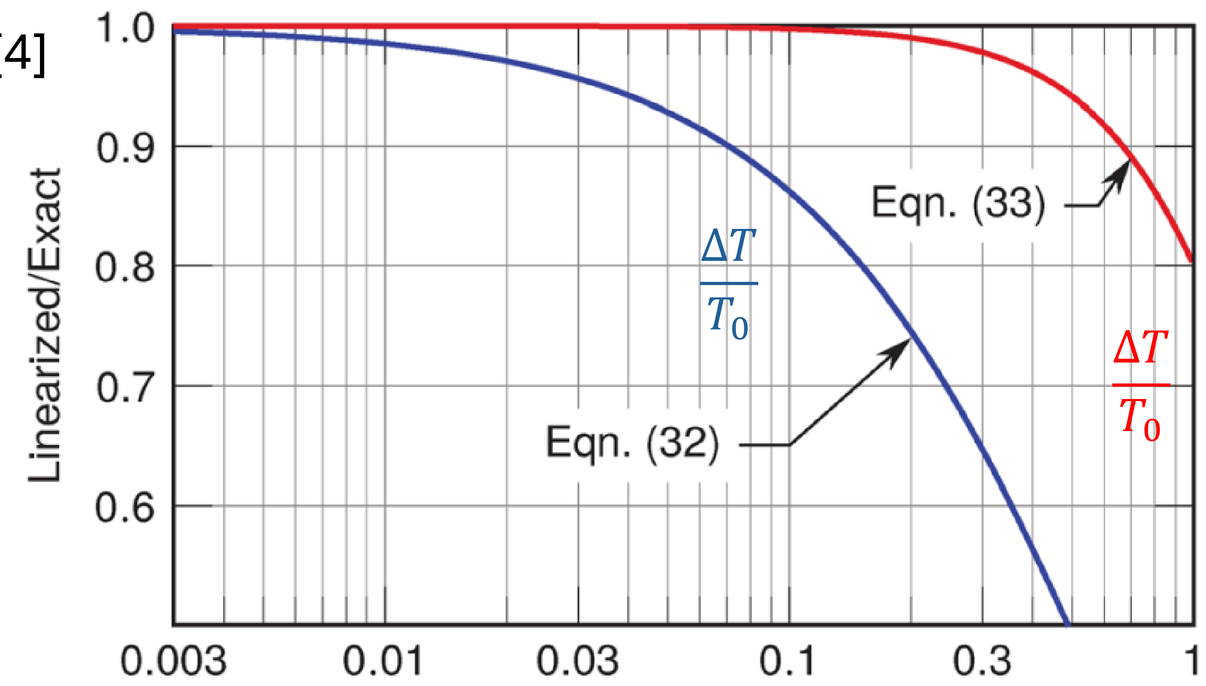
# State of the Art

- Linearization errors ratio due to $T_0$ [4]
- For : $T_0 = T_1$

$$\frac{4T_0^3 \Delta T}{T_1^4 - T_2^4} = \frac{4}{(1 + (\frac{T_2}{T_0})^2)(1 + \frac{T_2}{T_0})}$$

- For : $T_0 = \frac{T_1 + T_2}{2}$

$$\frac{4T_0^3 \Delta T}{T_1^4 - T_2^4} = \frac{1}{1 + \frac{1}{4}(\frac{\Delta T}{T_0})^2}$$

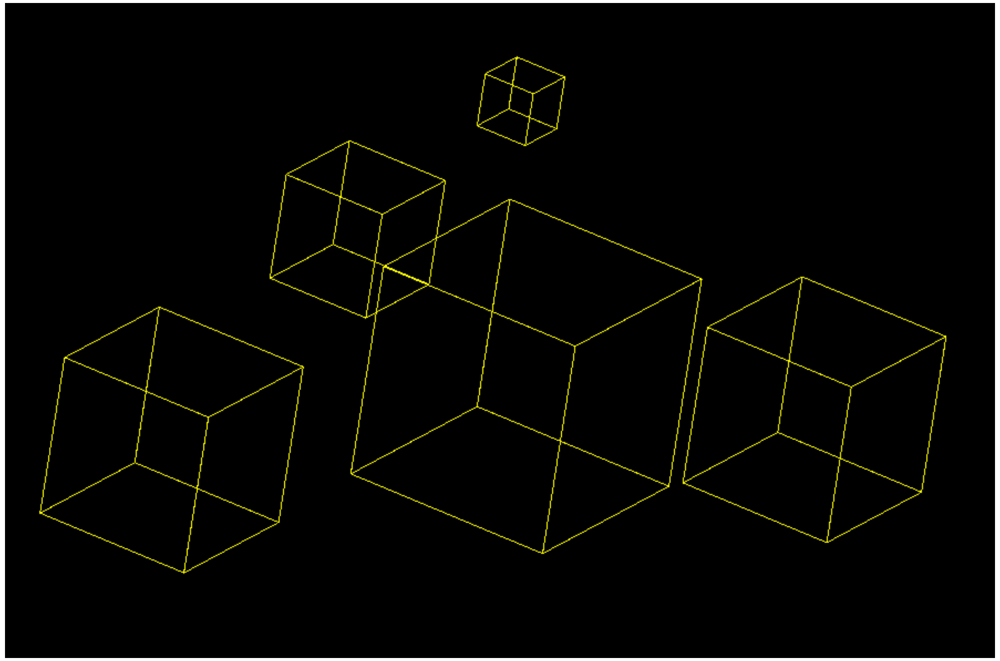



*Linearization error for a black surface about $T_0$*


[4] J. H. Lienhard V, "Linearization of Nongray Radiation Exchange: The Internal Fractional Function Reconsidered," *Journal of Heat Transfer*, vol. 141, p. 052701, 03 2019.

# Use case

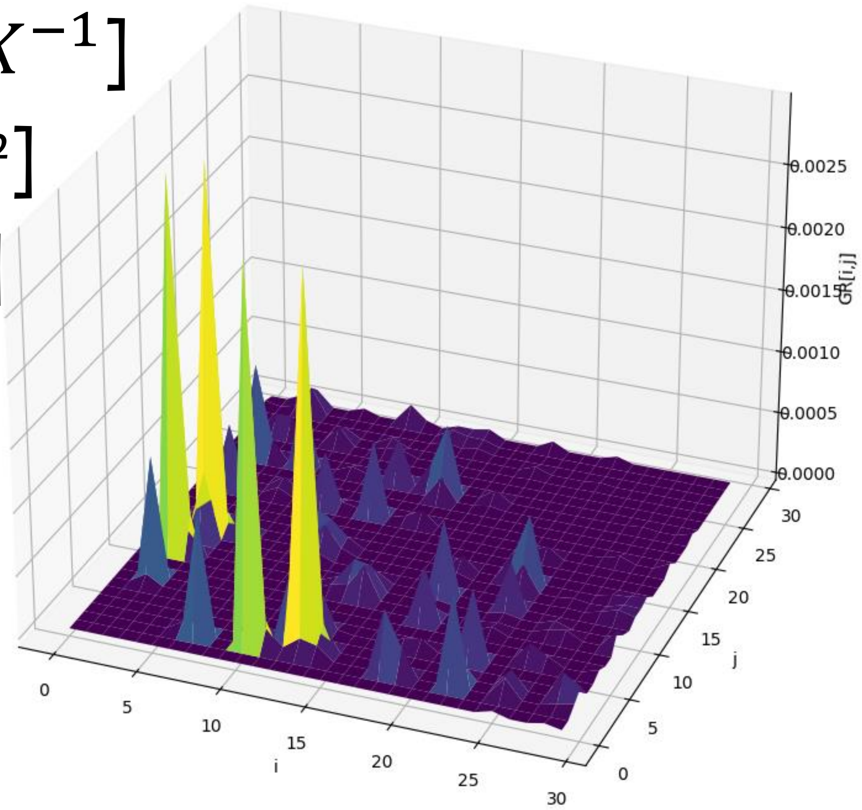
- e-Therm
- **Radiative** model
- 5 cubes, 1 node for each face -> 30 nodes
- **No** external fluxes
- **No** environment



Geometry of the 30 Nodes

# Use case

- Conductive couplings : $[50, 250]$ in $[W.K^{-1}]$
- Radiative couplings : $[0, 3 \cdot 10^{-3}]$ in $[m^2]$
- Heat capacitance : $[100, 250]$ in $[J.K^{-1}]$
- Same material
  - α = 0,2 (absorptivity)
  - ρ = 0,8 (reflectivity)
  - ε = 0,2 (emissivity)




*Visualization of the large disparities in the values of my radiative exchange coupling matrix with i and j the nodes indices*

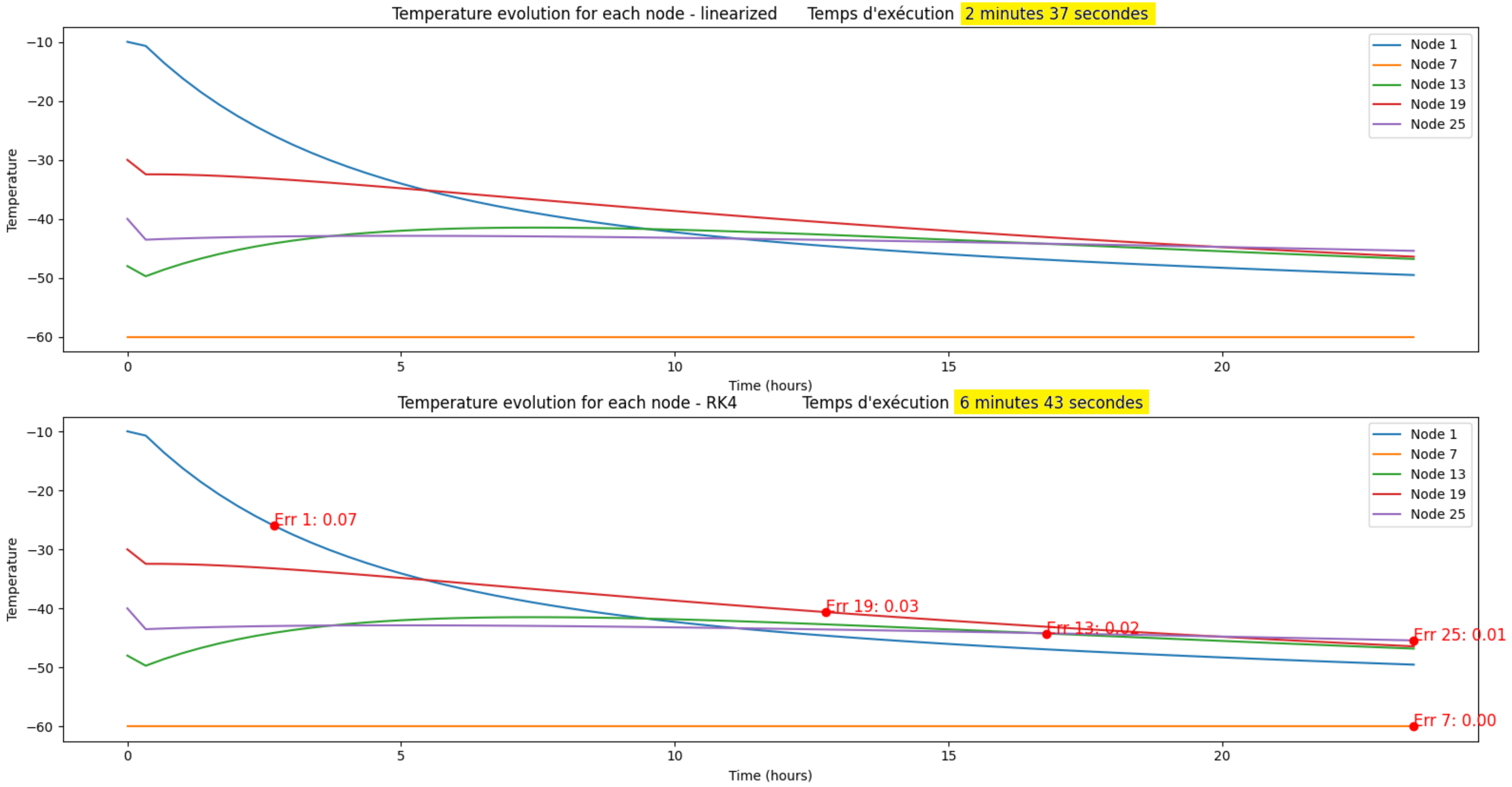
# Results (no internal power)

- Many parameters ($T_{init}$, h, $Q_i$, $GL_{ij}$...) -> multiple configurations

- For example, a case where there is **no internal power** ($Q_i$ = 0) :

**2min37** vs **6min43**
Max error **: 0.07°C**

*Temperature profile (without internal power) from my linearized equation versus that from my RK4 code*



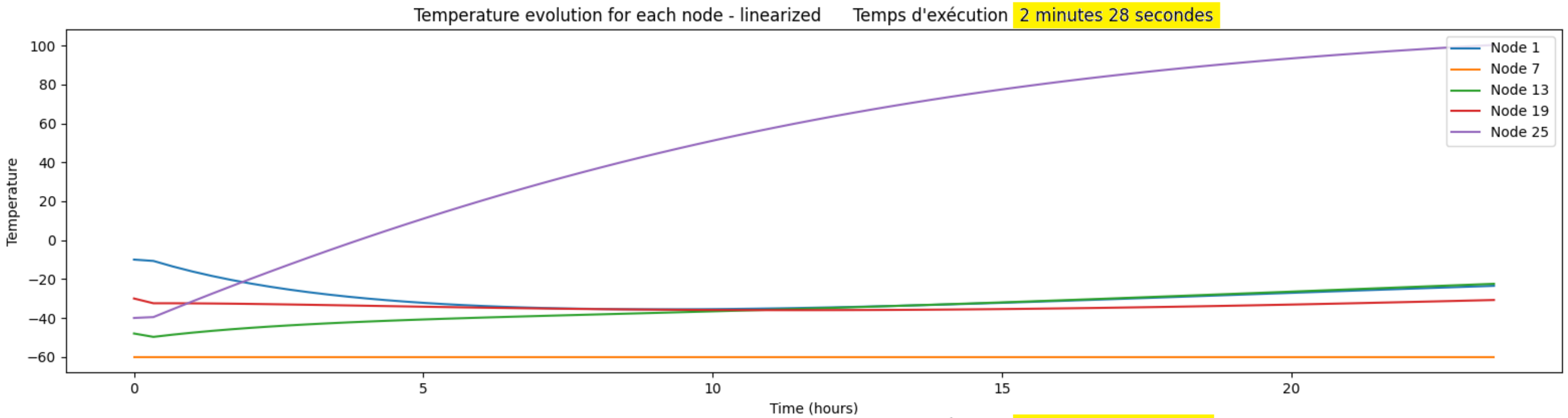
# Results (internal power)

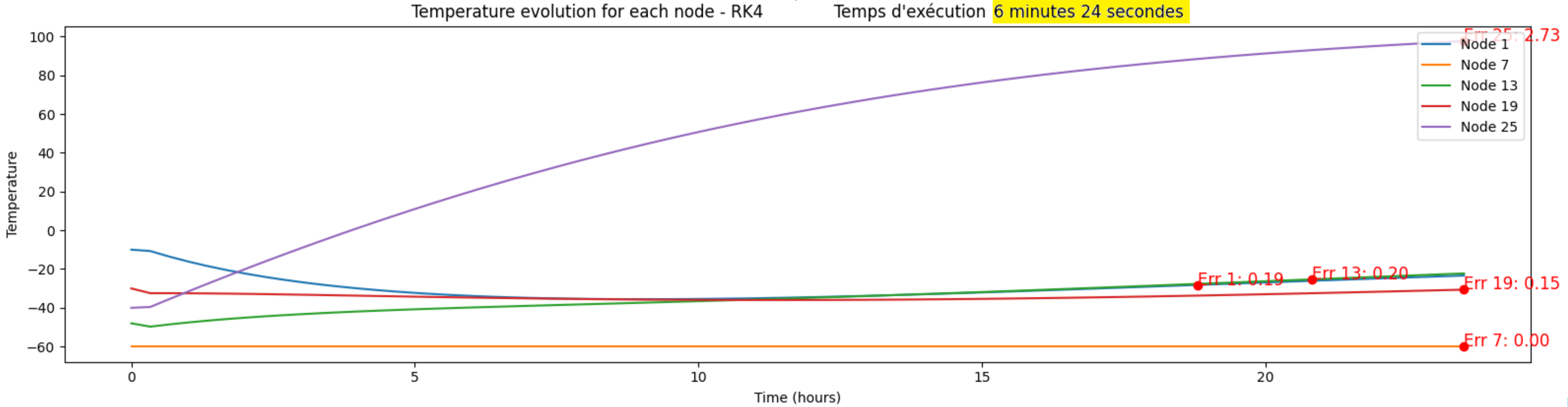
- Same configuration with $Q_i$ = 2 [W]

**2min28** vs **6min24**
Max error **: 2.73°C**

- The error **propagates** (or compensates)

*Temperature profile (with internal power) from my linearized equation versus that from my RK4 code*



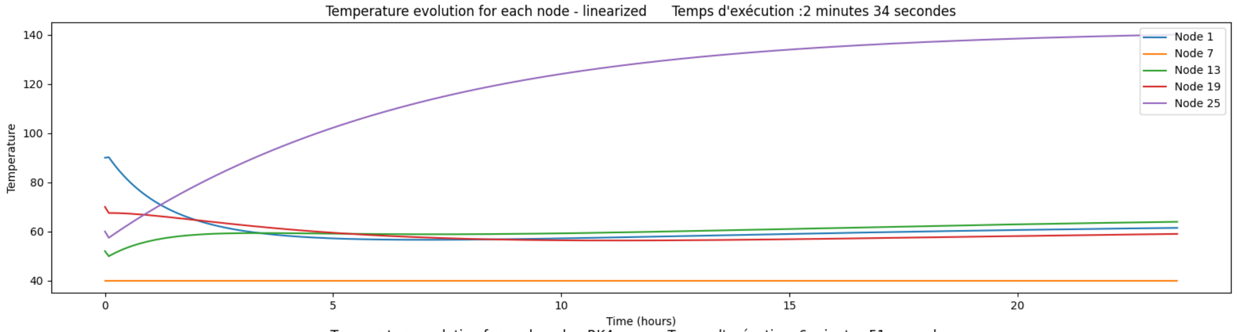



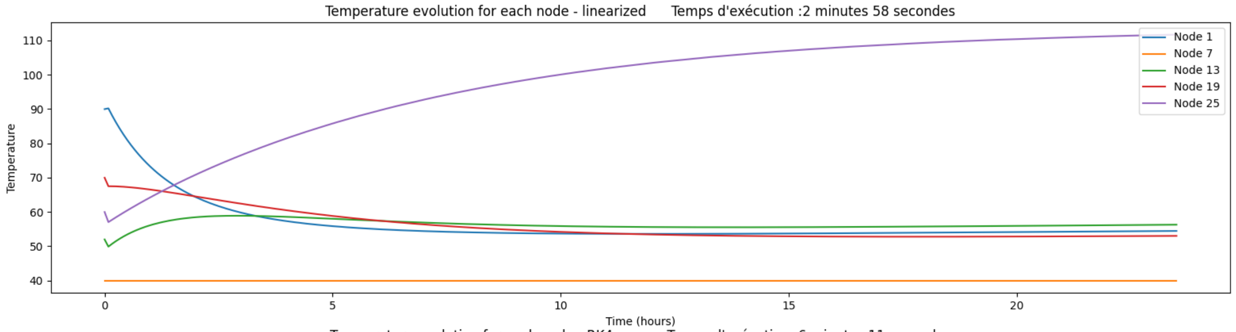
# Results (heat capacitance)

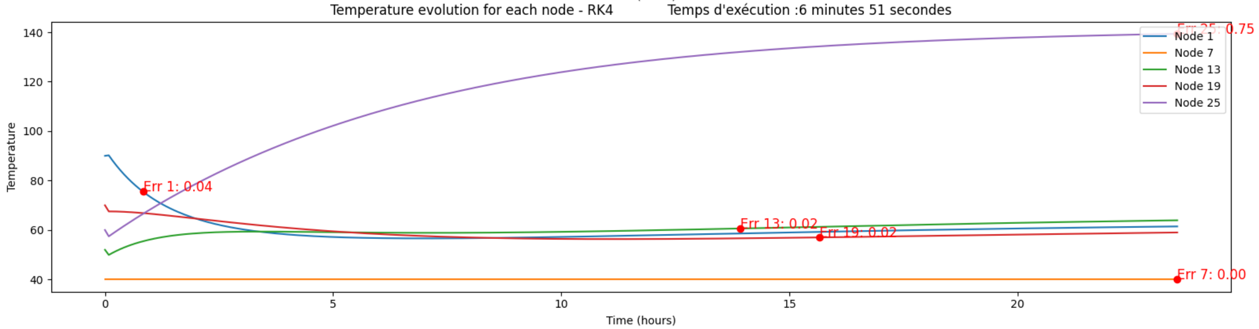
**2min34** vs **6min51**
Max error **: 0.75°C**

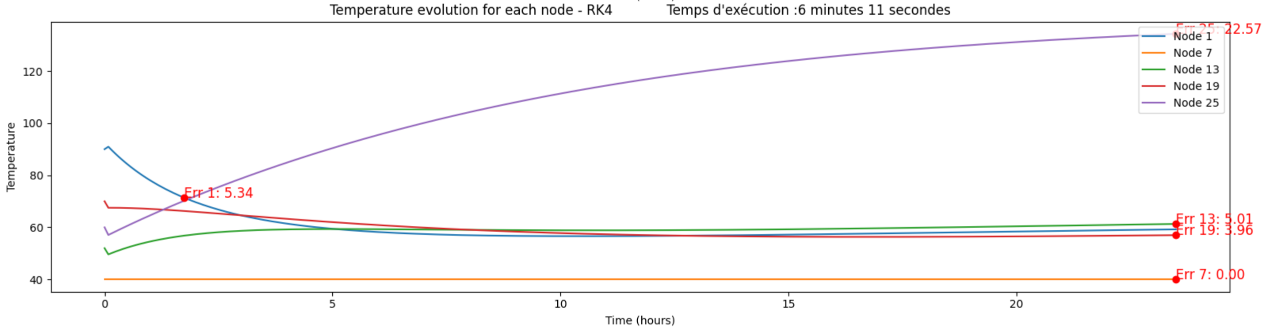
- Same configuration with internal power
- Cp = 100 $[J.K^{-1}]$
- Cp = 150 $[J.K^{-1}]$

**2min58** vs **6min11**
Max error **: 22.57°C**




- Linearization exponential solution does not reflect the real impact of Cp
- Cp does have an impact on RK4 stability scheme -> need to change integration step h

# Results (integration step)

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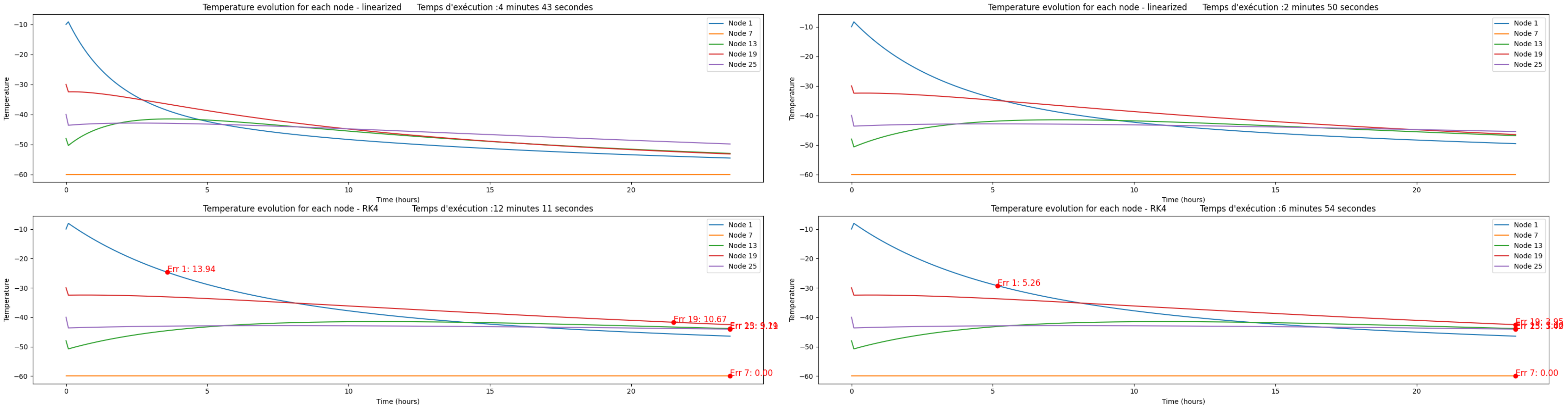
- Same configuration without internal power

**4min43** vs **12min11**
Max error : **13.94°C**

- **h =0.25**

- **h =0.5**

**2min50** vs **6min54**
Max error : **5.26°C**




- No difference for RK4 method
- 2x more time steps -> potentially 2x more errors due to linearization

# Conclusion

- Conclusion
  - Multiple iterations (86400sec) -> Error hard to predict
  - 3x faster than RK4 but needs validation on a real use case
- Future work :
  - Optimize integration step h
  - Try another linearization scheme
  - Parallelization of Runge-Kutta method [5]

[5] P. Van Der Houwen and B. Sommeijer, "Parallel iteration of high-order runge-kutta methods with stepsize control," *Journal of Computational and Applied Mathematics*, vol. 29, no. 1, pp. 111–127, 1990.

Thank you for your time !

Feel free to ask any questions or share your thoughts.