A New Statistical Approach to Refining the H-G Model for Near-Earth Asteroids

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EU-ESA Workshop on Size Determination of Potentially Hazardous Near-Earth Objects
ESA - ESOC

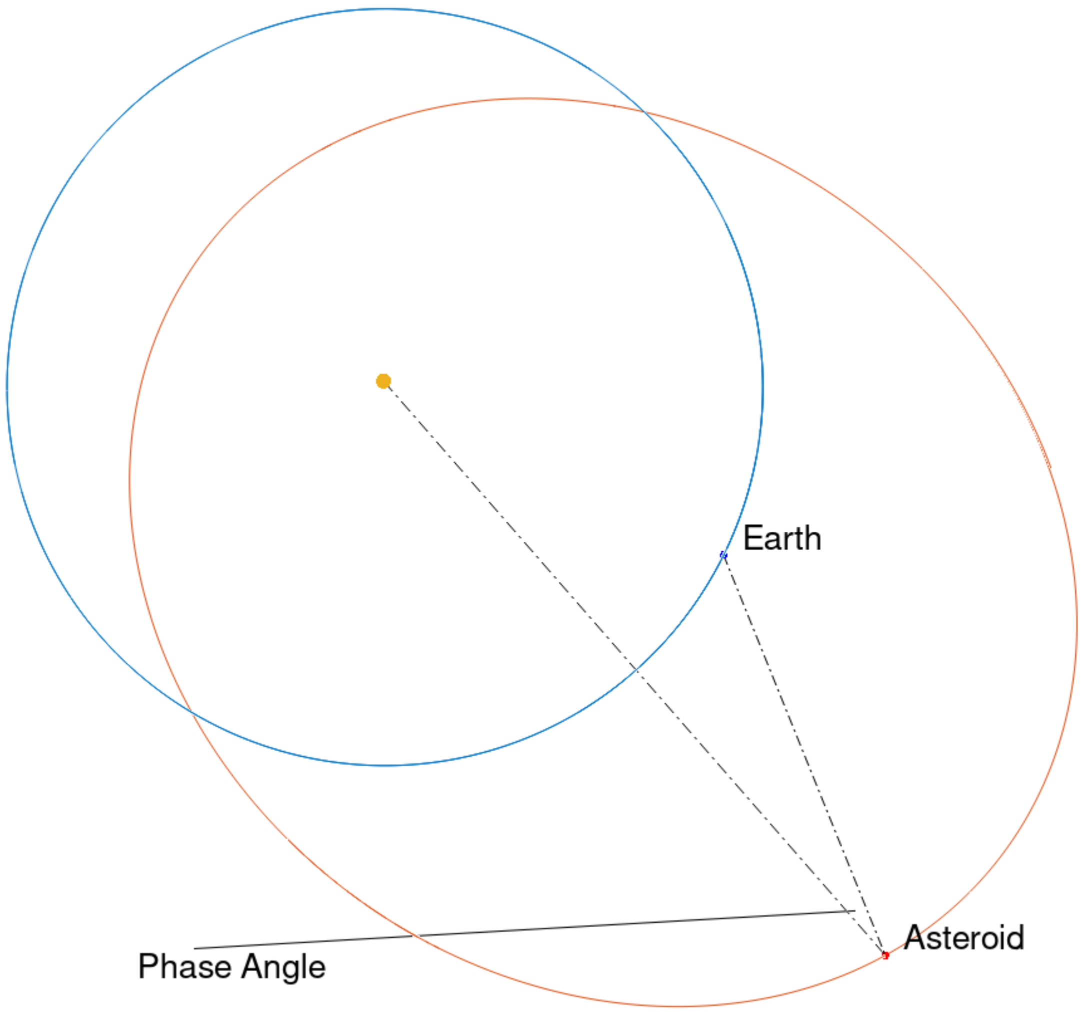
12/11/2024

What is the absolute magnitude?

Absolute magnitude H

Magnitude of an object:

- Located at 1 au from the Sun
- Located at 1 au from the observer
- Observed at 0 phase angle



Absolute magnitude models

$$H = m - 5\log_{10}(r \cdot \Delta) - g(\gamma)$$

```
m: visual magnitude
r: Asteroid-Sun distance
Δ: Asteroid-Observer distance
γ: phase angle
```

Bowell et al. 1989 - HG model

$$g(\gamma) = -2.5\log_{10}((1-G)\phi_1(\gamma) + G\phi_2(\gamma))$$

Muinonen et al. 2010 - HG1G2 model

$$g(\gamma) = -2.5\log_{10}(G_1\phi_1(\gamma) + G_2\phi_2(\gamma) + (1 - G_1 - G_2)\phi_3(\gamma))$$

Carry et al. 2024 - sHG1G2

$$g(\gamma, \alpha, \delta) = -2.5\log_{10}(G_1\phi_1(\gamma) + G_2\phi_2(\gamma) + (1 - G_1 - G_2)\phi_3(\gamma)) + s(\alpha, \delta)$$

H mag estimation implemented in Aegis

Aegis: ESA NEOCC Orbit Determination and Impact Monitoring System [1]

Fit with ***HG* model** by Bowell et al. 1989, using:

1) Slope parameter ***G* = 0.15**
2) Use **debiasing scheme** by [2]
3) Simple **weighting scheme** for photometry based on reported *V* precision
    1) σ = 1 if no decimal in *V*
    2) σ = 0.7 if 1 decimal is reported in *V*
    3) σ = 0.5 if 2 decimals are reported in *V*
4) Convert *V* into *H* and get nominal value by **weighted LS**
5) Outlier rejection scheme
6) RMS of the residuals

NOTE: The same model is used for computing V magnitude predictions in ephemerides

[1] Fenucci et al. 2024, *The Aegis Orbit Determination and Impact Monitoring System and services of the ESA NEOCC web portal*, CMDA
[2] Hoffmann et al. 2024, *Debiasing astro-Photometric Observations with Corrections Using Statistics (DePhOCUS)*, Icarus

H mag estimation implemented in Aegis

Aegis produces **Residuals Weights Observations** (rwo) files:

- Astrometric measurements and residuals
- Astrometric errors
- Photometric measurements and residuals
- Photometric errors

Available from the **NEOCC Web Portal**. Available in **ADES xml** format in **the future**.

```
version =    3
errmod  = 'vfcc17'
RMSast  =    8.14489E-01
RMSmag  =    3.58999E-01
END_OF_HEADER
! Object   Obser ============= Date ============= ================== Right Ascension =================  ================= Declination ===================== ==== Magnitude ==== Ast Obs  Residual SEL
! Design   K T N YYYY MM DD.dddddddddd   Accuracy HH MM SS.sss  Accuracy      RMS  F     Bias      Resid sDD MM SS.ss  Accuracy      RMS  F     Bias      Resid Val  B   RMS  Resid Cat Cod       Chi A M
 2023VD3   O C 1 2023 11 06.40242       1.000E-05 04 05 38.370  1.500E-01    0.500 F    0.000      0.161 +00 12 59.80  1.000E-01    0.500 F    0.000     -0.120 19.8 G  0.70  -0.12   V G96      0.41 1 1
 2023VD3   O C 1 2023 11 06.40761       1.000E-05 04 05 41.790  1.500E-01    0.500 F    0.000     -0.098 +00 13 07.50  1.000E-01    0.500 F    0.000     -0.092 19.5 G  0.70  -0.42   V G96      0.28 1 1
 2023VD3   O C 1 2023 11 07.393635      1.000E-06 04 27 43.580  1.500E-02    0.800 F    0.000      0.225 +00 55 15.20  1.000E-02    0.800 F    0.000      0.377 19.20G  0.50   0.36   V 703      0.55 1 1
 2023VD3   O C 1 2023 11 07.400733      1.000E-06 04 27 58.295  1.500E-02    0.500 F    0.000     -0.163 +00 55 45.59  1.000E-02    0.500 F    0.000     -0.004 18.59G  0.50  -0.24   V G96      0.34 1 1
 2023VD3   O C 1 2023 11 07.404242      1.000E-06 04 28 05.670  1.500E-02    0.800 F    0.000      1.042 +00 56 00.80  1.000E-02    0.800 F    0.000     -0.484                     V 703      1.45 1 1
 2023VD3   O C 1 2023 11 07.405907      1.000E-06 04 28 09.124  1.500E-02    0.500 F    0.000     -0.062 +00 56 08.34  1.000E-02    0.500 F    0.000     -0.048 18.67G  0.50  -0.16   V G96      0.16 1 1
 2023VD3   O C 1 2023 11 07.409560      1.000E-06 04 28 16.750  1.500E-02    0.800 F    0.000     -0.551 +00 56 24.10  1.000E-02    0.800 F    0.000     -0.725                     V 703      1.15 1 1
```

Weighted RMS of residuals

Photometry details

Acceptance flag

H mag estimation implemented in Aegis

→ 2023VD3 help

Last update: 2024-11-07 13:00 UTC

Summary | Orbit Properties | Physical Properties | Observations | Ephemerides | Close Approaches | MOID | Possible Impacts

OBSERVATIONAL INFORMATION

Arc Information	Value	Unit	Source
Arc Length	2	d	[1]
Unobserved	365	d	[1]
Normalized RMS of residuals	0.814489	-	[1]

ASTROMETRY SUMMARY

Optical	Value	Unit	Source
First Observation	2023-11-06.40242	-	[2]
Last Observation	2023-11-08.660703	-	[2]
Total Optical Observations (Discarded)	90 (4)	-	[1]

RADAR SUMMARY

Radar	Value	Unit	Source
First Observation	-	-	[3]
Last Observation	-	-	[3]
Delay (Discarded)	0 (0)	-	[1]
Doppler (Discarded)	0 (0)	-	[1]
Total Radar Observations (Discarded)	0 (0)	-	[1]

→ **DOWNLOAD ASCII FILE**

Note: The ASCII file is in a format suitable for Orbfit input.

Documentation

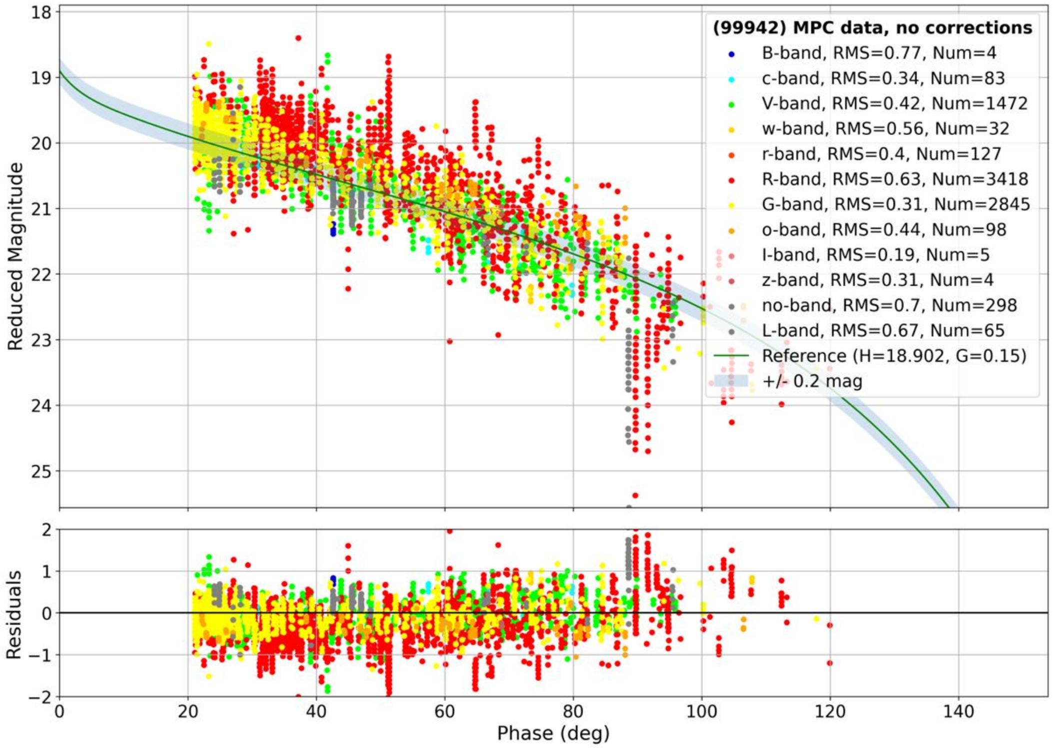
Observations Tab

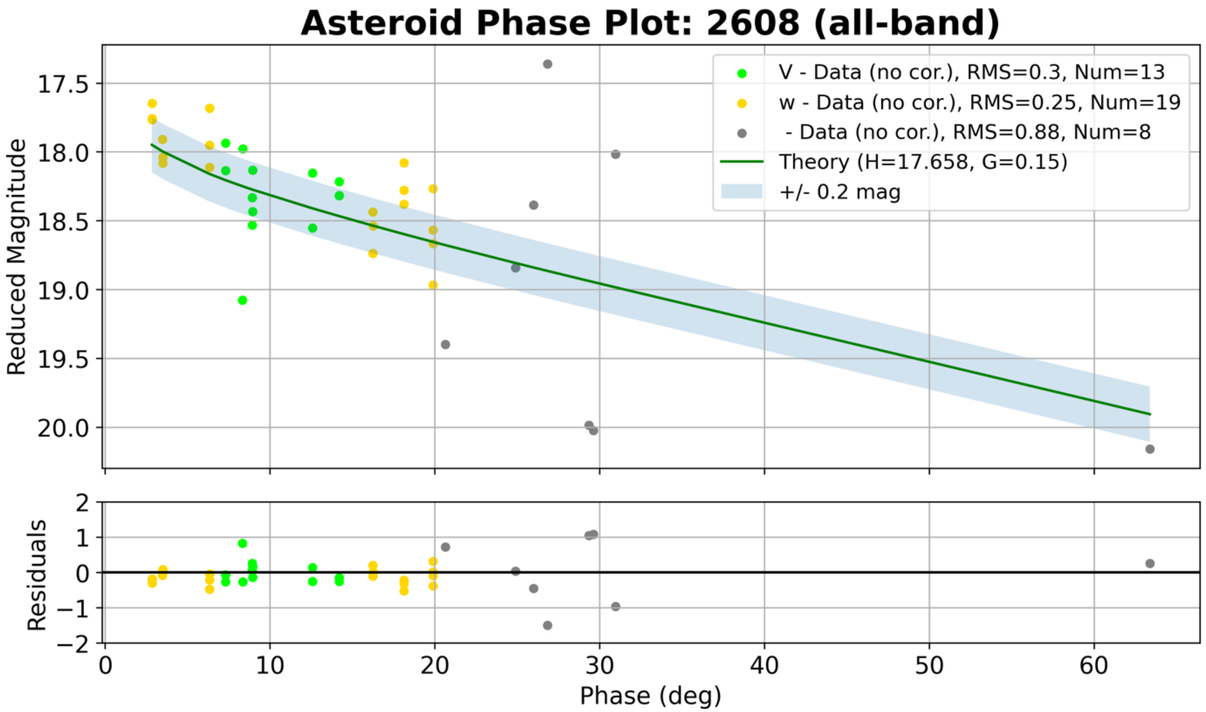
rwo file

https://neo.ssa.esa.int/

Some examples

It is often **not possible** to fit **H and G together -** especially for NEAs





Opened issues, motivation and goals of this work

Motivations:

1) Reliable determination of H is **crucial for impact assessment**
2) Even more important in the first phases during **discovery** and first **follow-up**

Opened issues with NEAs:

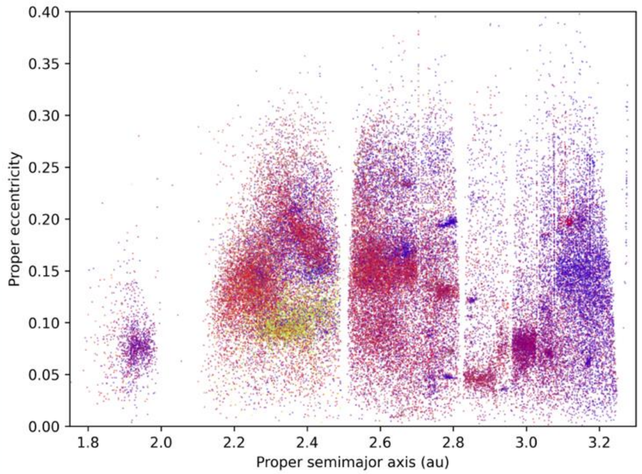
1) NEAs are generally observed a **high phase angles**
2) At discovery, with a short arc, the phase angles coverage is **even smaller**
3) Because of lightcurve effect, H could have **large errors**
4) Impossible to fit the slope parameter G
5) Small amount of data for other H models

Goal of this work:

1) More robust determination of H of NEAs using **statistical approach**

Statistical model for H estimation: Previous Works

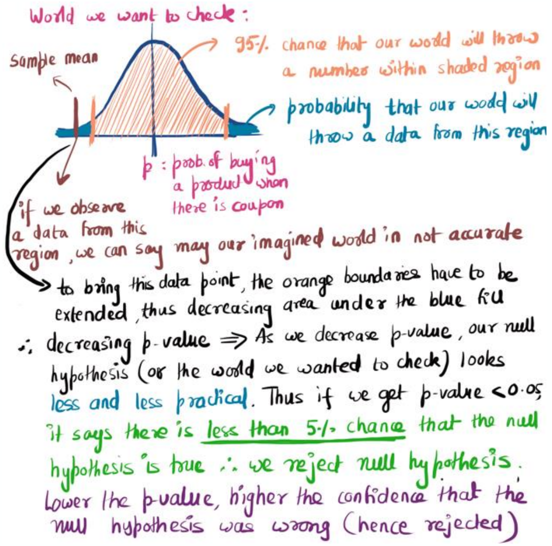
Step 1
Population models for:
- Colors
- Slope parameter
- Rotation period
- Phase curve



Step 2
- 500 synthetic asteroids
- Simulate V mag
- H residuals
- Minimize χ^2

$$\chi^2_{k,obs} = \sum_{j=1}^{n} \left[\frac{m_k(t_j; H'_k, G'_k, \theta'_k) - m(t_j)}{\delta m(t_j)} \right]^2$$

Step 3
- Compute uncertainties for H and G
- Compute errors



Vereš et al. 2015, *Absolute magnitudes and slope parameters for 250,000 asteroids observed by Pan-STARRS PS1 – Preliminary results*, Icarus 261

Statistical model for H estimation of NEAs

Main argument: G is generally different from 0.15

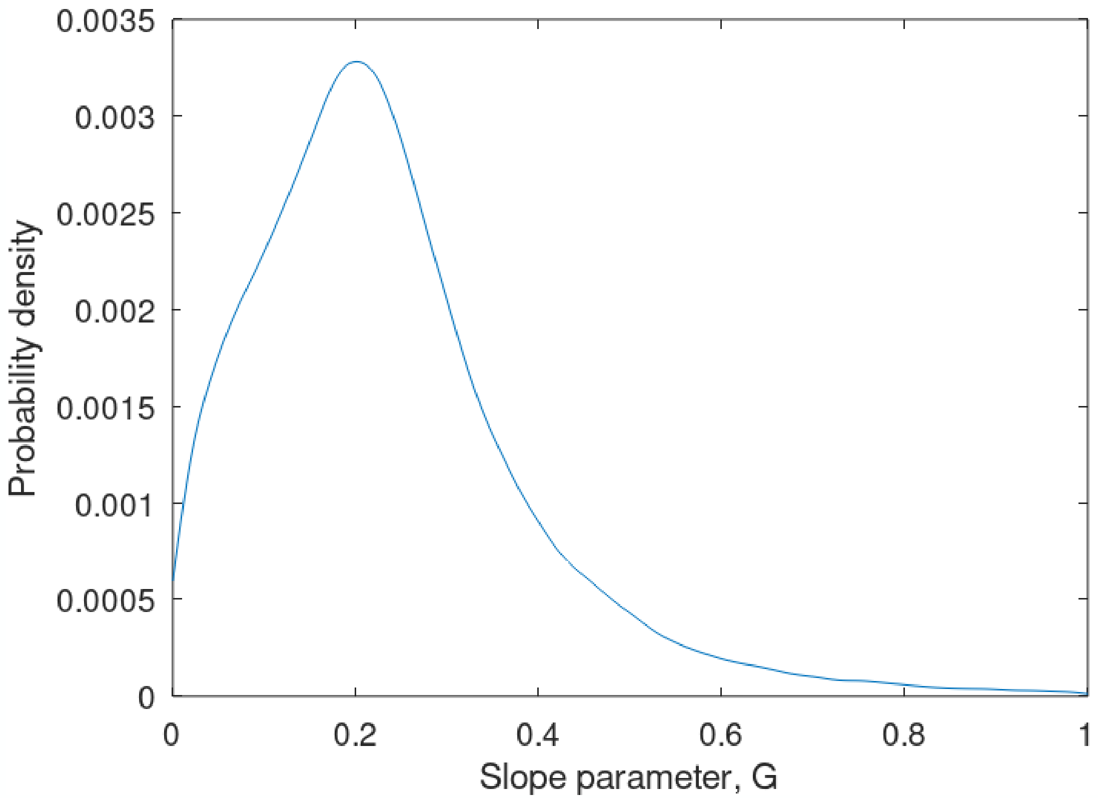


Fig. 1. Distribution of G determined from SsODNet [1].

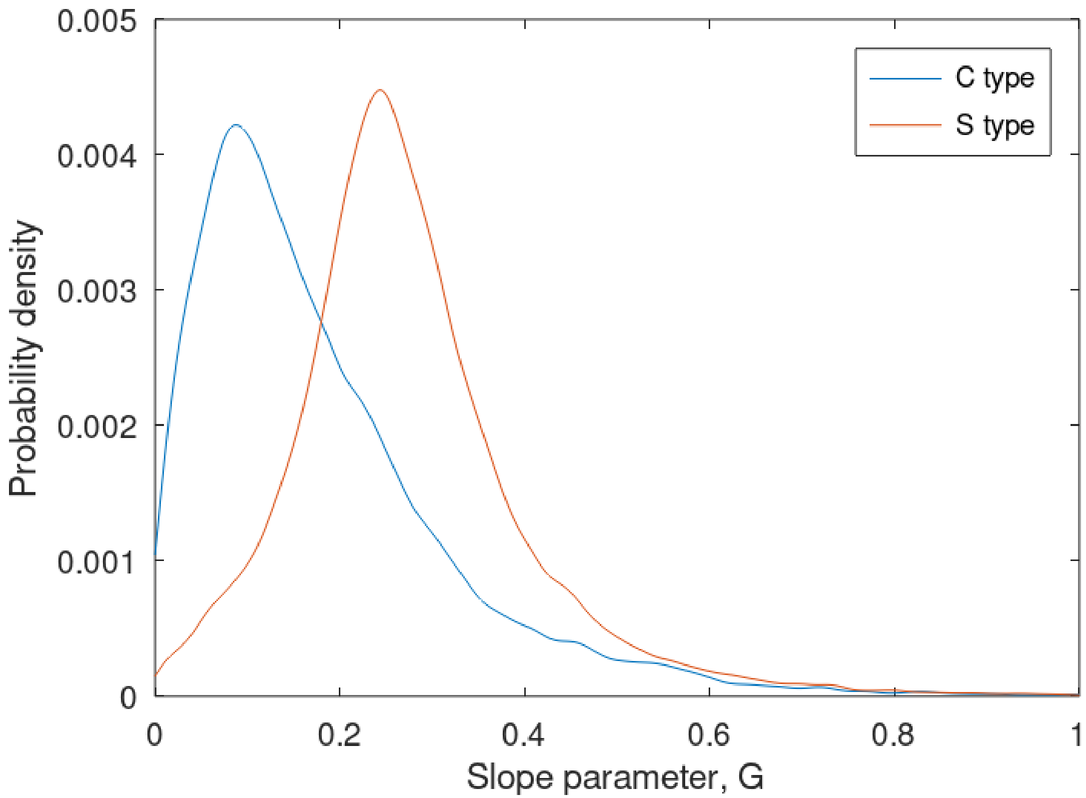


Fig. 2. Distribution of G of C- and S-type asteroids, determined from SsODNet [1].

[1] Berthier et al. 2023, *SsODNet: Solar system Open Database Network*, A&A 671, A151

Statistical model for H estimation of NEAs

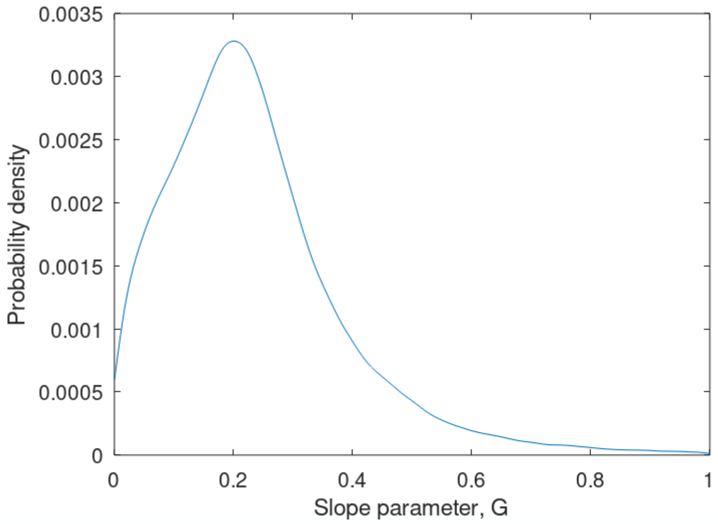
Monte Carlo based approach:

Step 1: A-priori population-based distribution of G

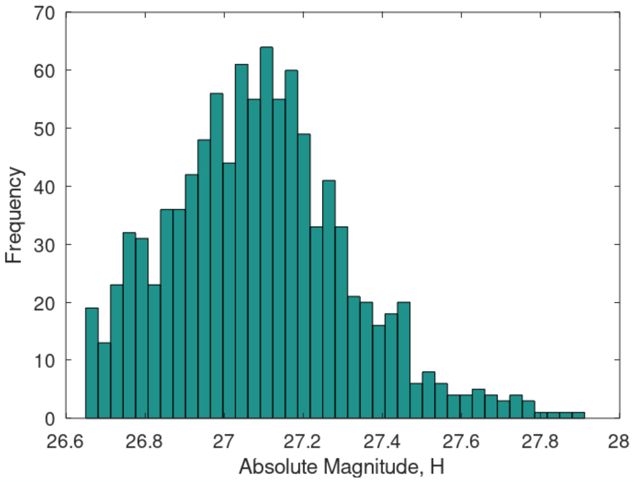
Step 2: Fit H over a sample of G

Step 3: Get statistics of H

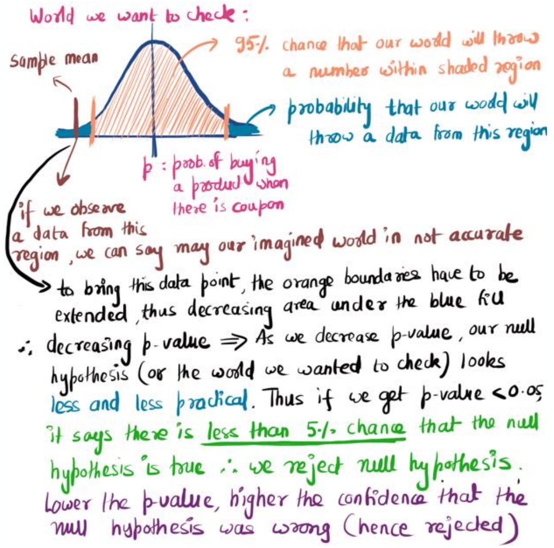
STEP 1



STEP 2



STEP 3



Example: 2023 VD3

2023 VD3: small NEA ranked 1st in NEOCC Risk List – 2 days arc – phase [20, 40] deg

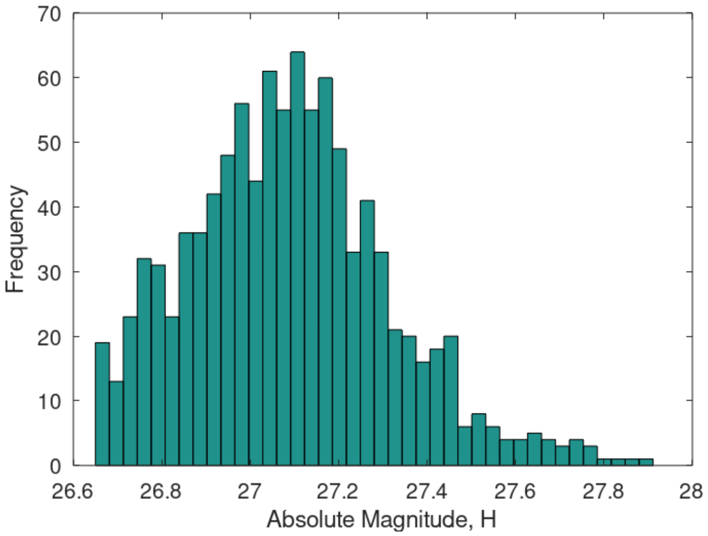


Fig. 1. Distribution of H.

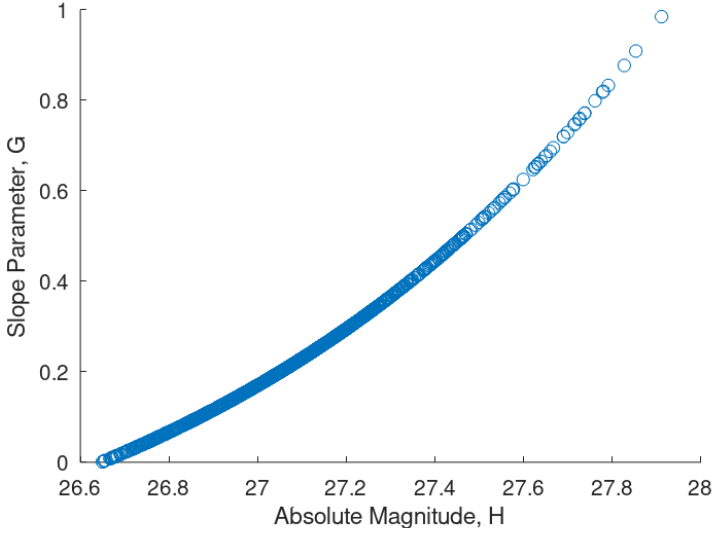


Fig. 2. Magnitude H vs. slope G.

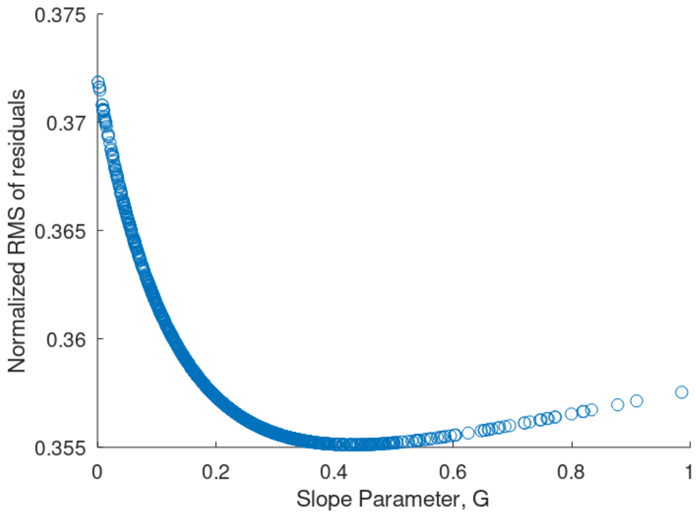


Fig. 3. Slope G vs. RMS H residuals.

Note: one-to-one dependency in H-G

The PDF of H can be computed **semi-analytically**:

$$f_Y(y) = f_X\left(g^{-1}(y)\right)\left|\frac{d}{dy}\left(g^{-1}(y)\right)\right|$$

Example: 2023 VD3

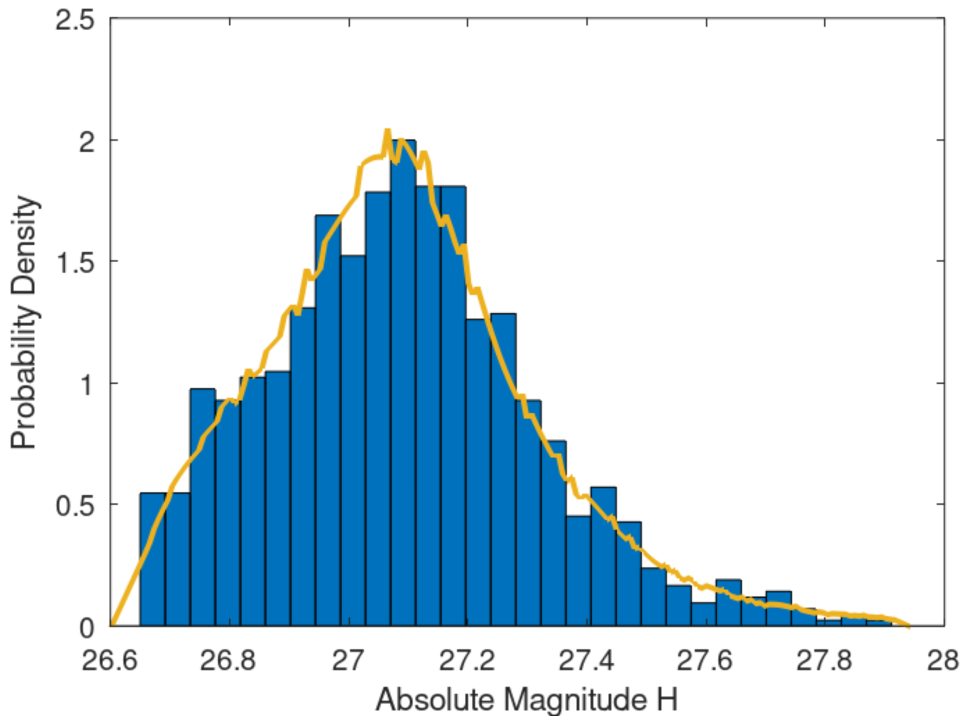


Fig 1. PDF of H from MC (histogram) and from semi-analytical model.

Estimated absolute magnitude:

$$H = 27.074^{+0.222}_{-0.212}$$

We can take also 5th and 95th percentile:

5th percentile: 26.755

95th percentile: 27.489

Comparisons with known asteroids

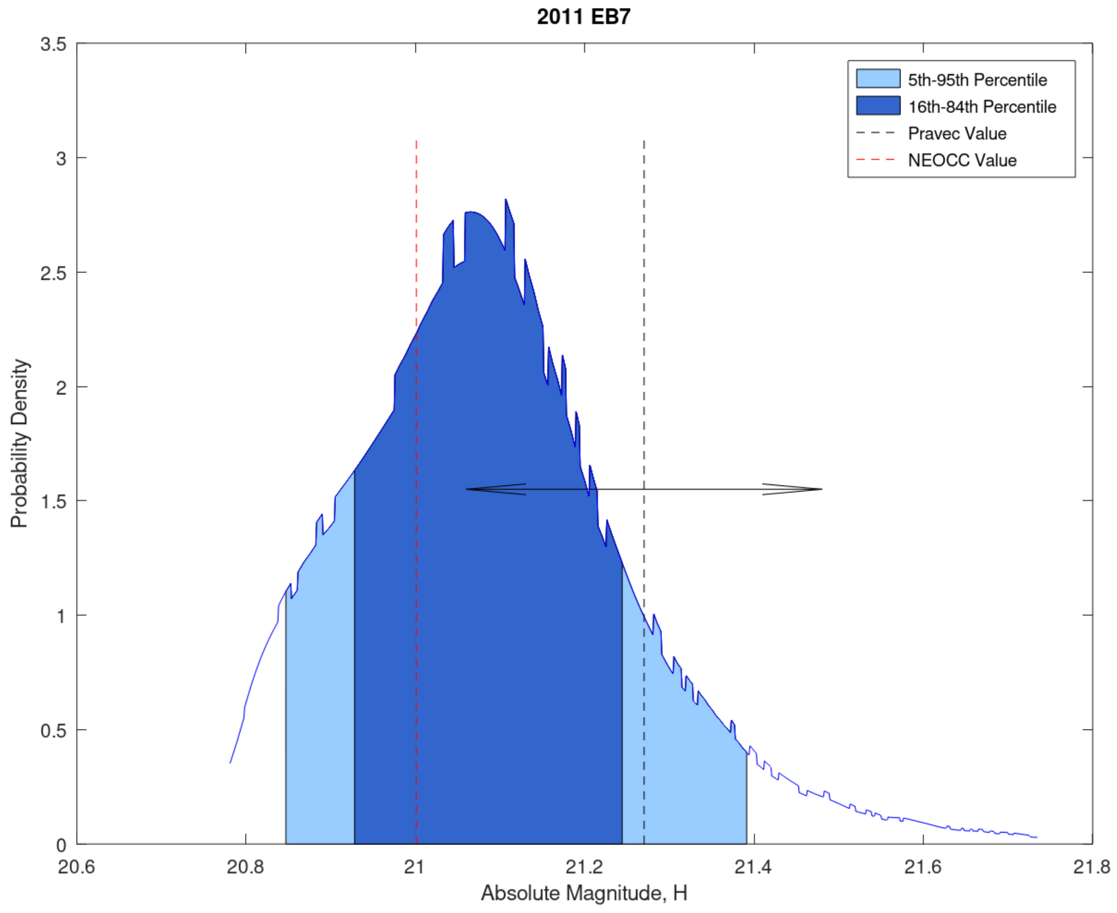
Sample set: asteroids from Pravec et al. 2012

Caveats:

1) We are **not fitting G**
2) We are **not solving for rotational state**
3) We use **only MPC data** (no high quality lightcurves)
4) The problem we are solving is different

[1] Pravec et al. 2012, *Absolute magnitudes of asteroids and a revision of asteroid albedo estimates from WISE thermal observations.* Icarus 221, 365–387

Example: 2011 EB7



Estimated from our model:

$$H = 21.08^{+0.165}_{-0.1652}$$

Estimated from Pravec et al. 2012:

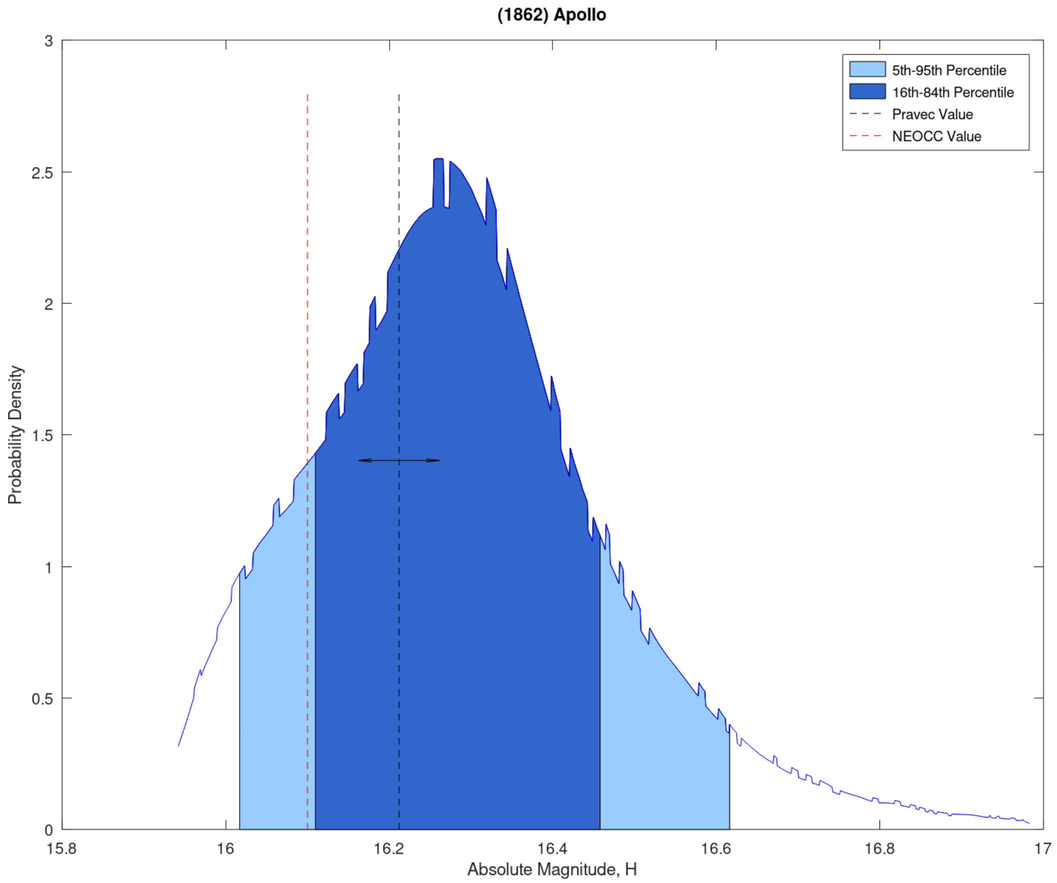
$$H = 21.27^{+0.21}_{-0.21}$$

Assumed slope

$$G = 0.24^{+0.11}_{-0.11}$$

[1] Pravec et al. 2012, *Absolute magnitudes of asteroids and a revision of asteroid albedo estimates from WISE thermal observations.* Icarus 221, 365–387

Example: (1862) Apollo



Estimated from our model:

$$H = 16.27^{+0.169}_{-0.180}$$

Estimated from Pravec et al. 2012:

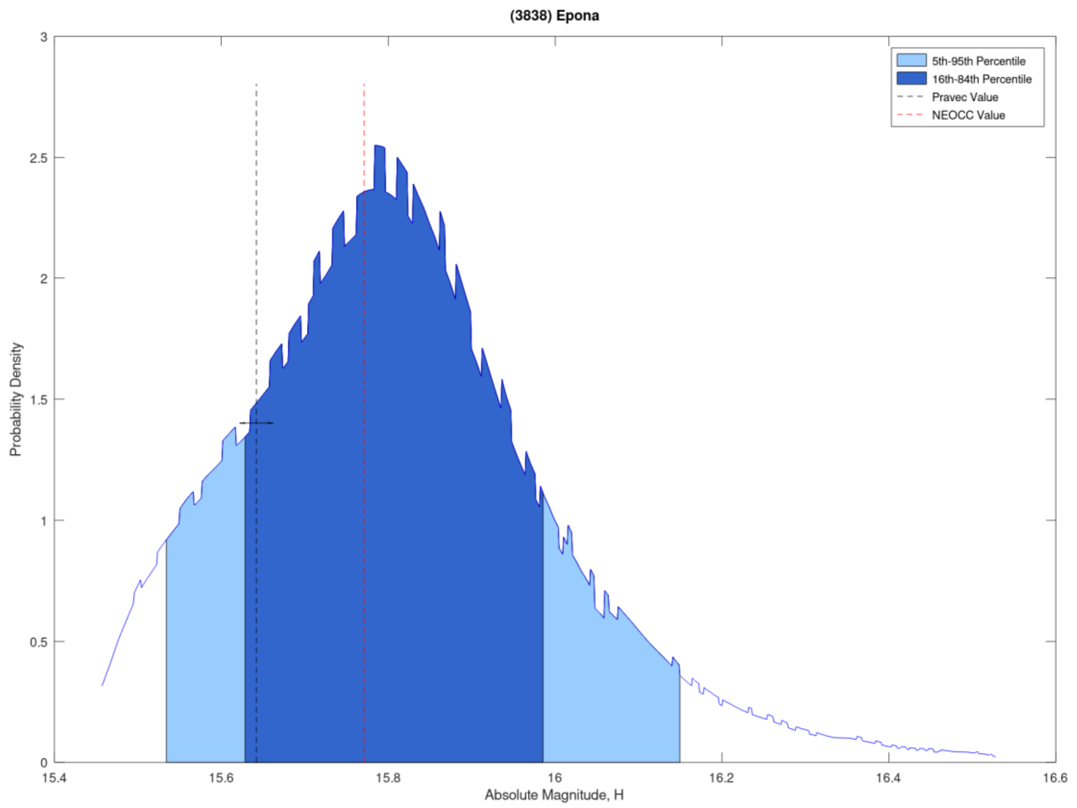
$$H = 16.21^{+0.05}_{-0.05}$$

Assumed slope

$$G = 0.24^{+0.11}_{-0.11}$$

[1] Pravec et al. 2012, *Absolute magnitudes of asteroids and a revision of asteroid albedo estimates from WISE thermal observations.* Icarus 221, 365–387

Example: (3838) Epona



Estimated from our model:

$$H = 15.80^{+0.185}_{-0.172}$$

Estimated from Pravec et al. 2012:

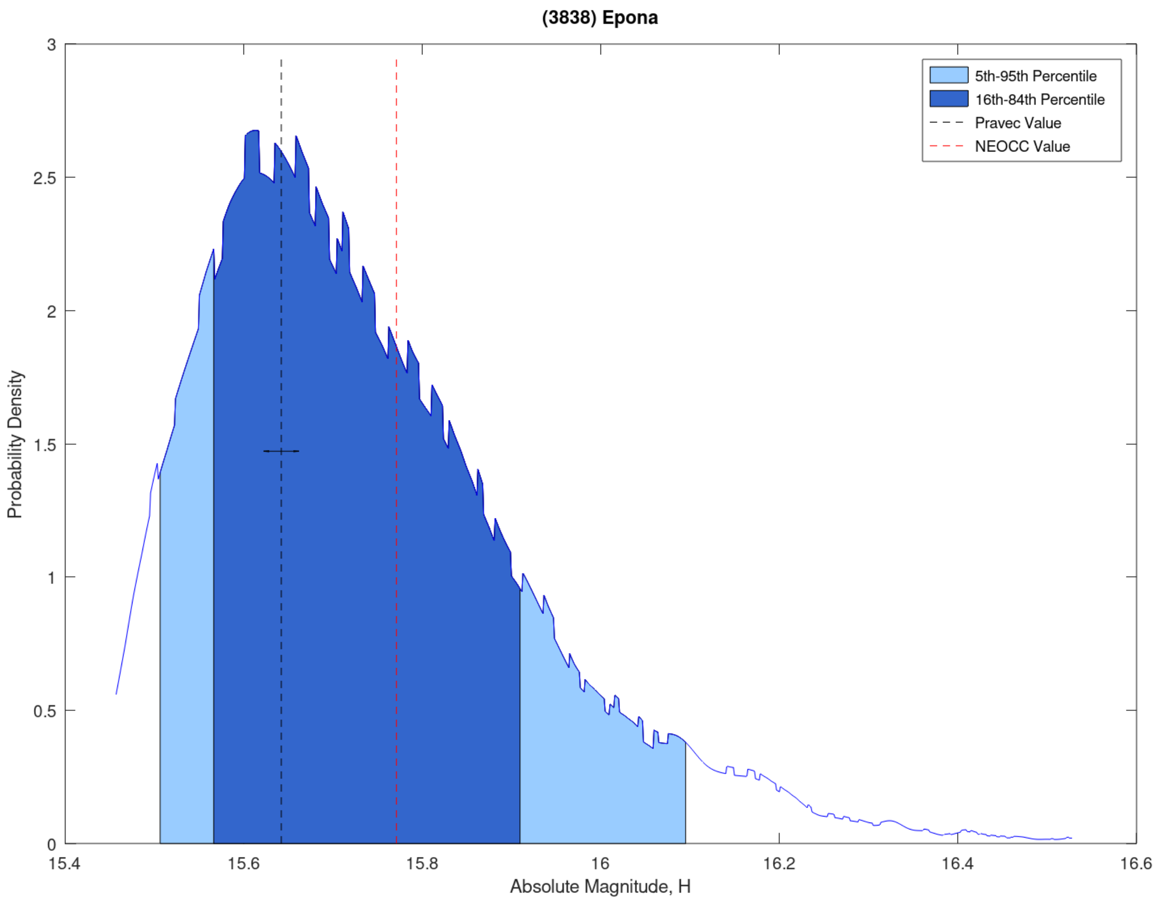
$$H = 15.642^{+0.02}_{-0.02}$$

Estimated slope

$$G = 0.05^{+0.01}_{-0.01}$$

[1] Pravec et al. 2012, *Absolute magnitudes of asteroids and a revision of asteroid albedo estimates from WISE thermal observations.* Icarus 221, 365–387

Example: (3838) Epona



Epona is a C type asteroid (SDSS colors)

Estimated from our model:

$$H = 15.70^{+0.206}_{-0.138}$$

Estimated from Pravec et al. 2012:

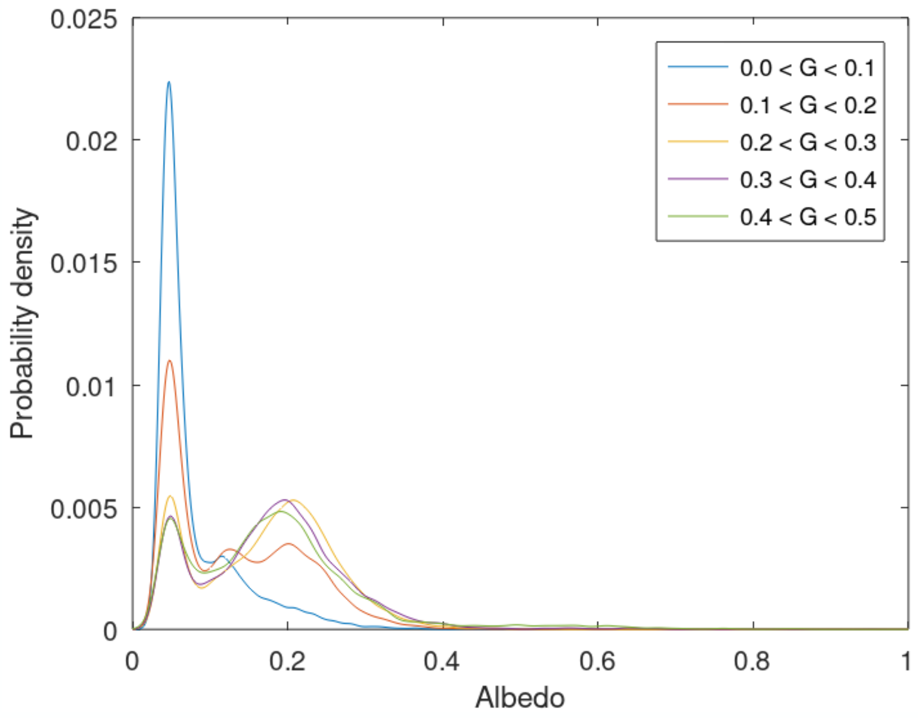
$$H = 15.642^{+0.02}_{-0.02}$$

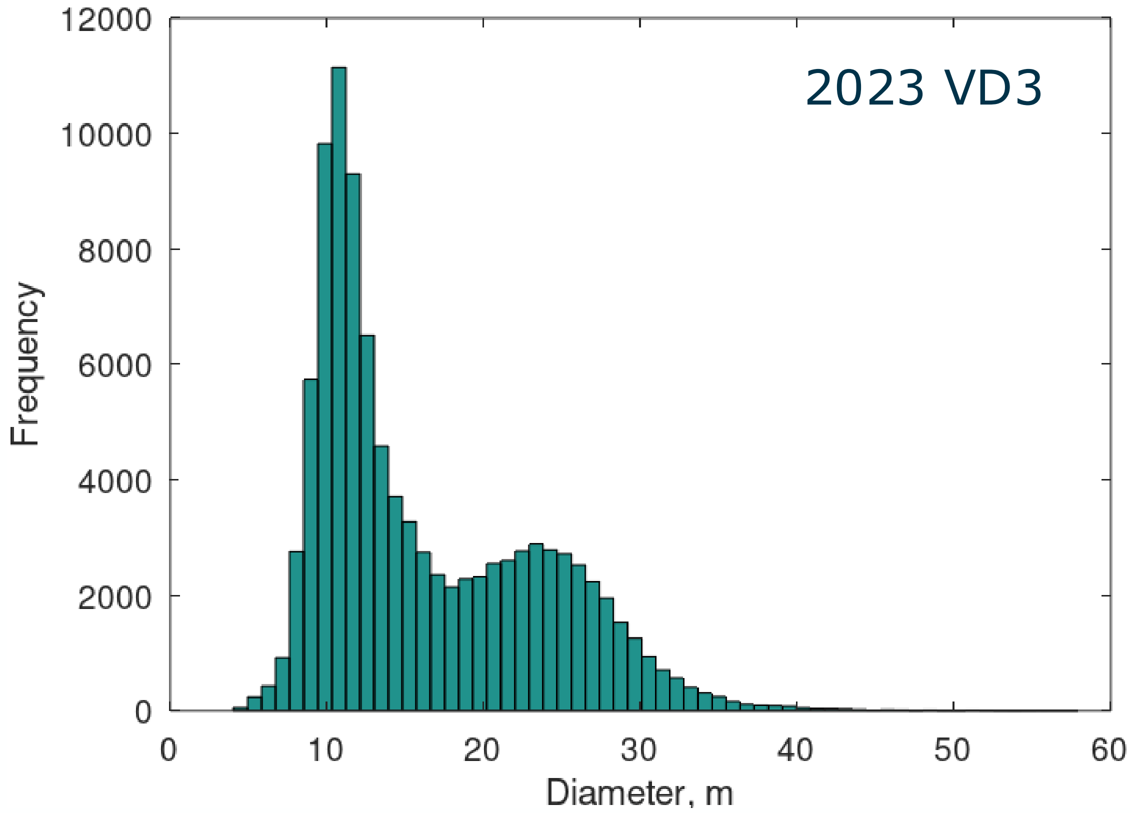
[1] Pravec et al. 2012, *Absolute magnitudes of asteroids and a revision of asteroid albedo estimates from WISE thermal observations.* Icarus 221, 365–387

Extrapolation of diameter distribution

NOTE: Different G have different albedos.

Diameter distribution: Monte Carlo method to convert (H, G) into D





Future works and improvements

Grocery list:

- **Debiasing of G distribution**
- Use full **population model of NEOs** (e.g. Granvik et al. 2018):
    - Extrapolate source region probabilities from orbital elements
    - Produce a debiased G distribution for each source region
    - Produce a G distribution according to source region probabilities
    - Follow the same procedure described here
- Use available **data about physical properties** (e.g. spectrum, colors, etc)
- Use of **other models of absolute magnitude**, e.g. Muinonen et al. 2011, Carry et al. 2024
    - What about the G12*? Could improve the fit for NEAs?
- Use of **ADES** for weighting of photometry
- Better **weighting scheme** for old photometry
- Could it help to use **lightcurve data** to improve the model?
- Comparisons with all the well known asteroids from Pravec et al. 2012