The Physics of Asymptotic High-Speed Flow Regimes II: Radiative Heat Transfer

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Introduction

Why model radiation?

- At sufficient high velocities radiation becomes the most important mode of heat transfer.
- Radiative effects cool flowfield (endothermic process) - contribution to asymptotic temperature limit

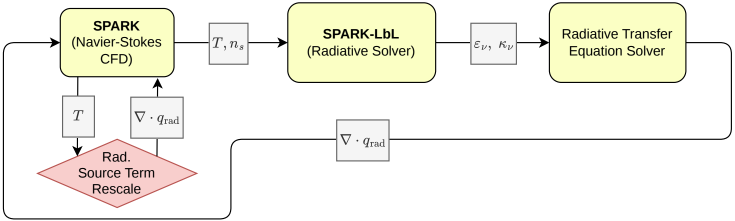
In this work:

- Implementation of radiative transfer models in CFD
- Focus on spherical geometries (meteor entry)
- Earth atmospheric composition
- Speeds ranging between $16 - 25\mathrm{km/s}$

Methods

Computational tools:

- SPARK hypersonic non-equilibrium CFD solver
- SPARK Line-by-Line radiation solver
- Radiative transfer equation solver



Weakly coupled approach - high fidelity calculations of ε_ν and κ_ν are not made during the CFD runs. The radiative source term is updated according to simplified physical models.

Case studies of increasing complexity

Increasing complexity

1 – Optically thin

No reabsorption of radiation

2 – Absorption is considered

Reabsorption of radiation
using the tangent-slab model

3 – Addition of precursor heating

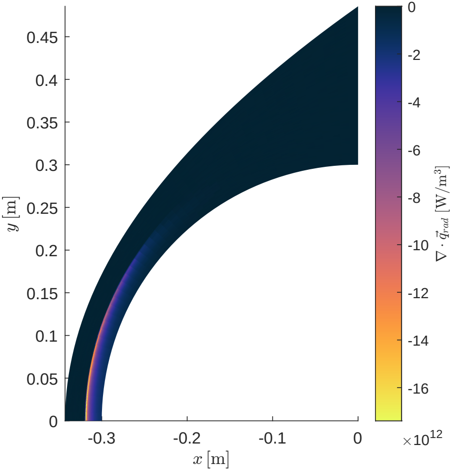
Precursor gas heating effects

Optically Thin Model - 25km/s (second version)

All emitted radiation escapes the domain (no reabsorption).

$$\begin{aligned}\nabla \cdot q_{\mathrm{rad}} &= -4\pi \int \varepsilon_\nu \, \mathrm{d}\nu \\ &= -4\pi \frac{\int \kappa_\nu B_\nu d\nu}{\int B_\nu d\nu} \int B_\nu d\nu \\ &\equiv -4\kappa_{\mathrm{Pl}} \sigma T^4 \\ &\approx -4\kappa_{\mathrm{Pl}}^{\mathrm{initial}} \sigma T^4\end{aligned}$$

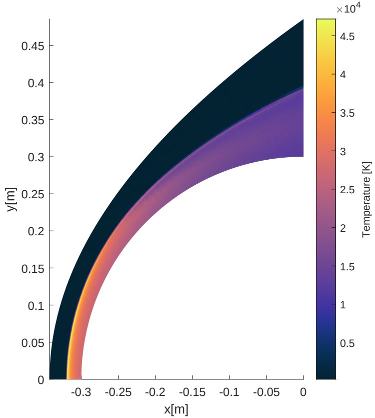
Source term expressed as a function of initial Planck opacity ($\kappa_{\mathrm{Pl}}^{\mathrm{initial}}$), yielding a T^4 dependence.



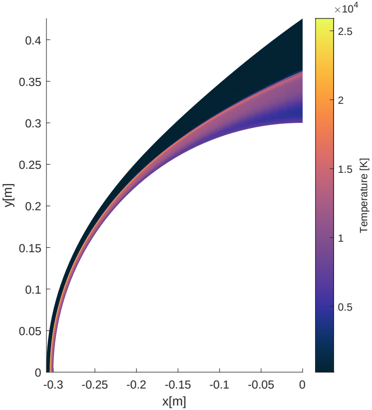
Initial radiative source term intensity

Optically Thin Model - 25km/s

The optically thin assumption yields nonphysical radiative heat source terms.



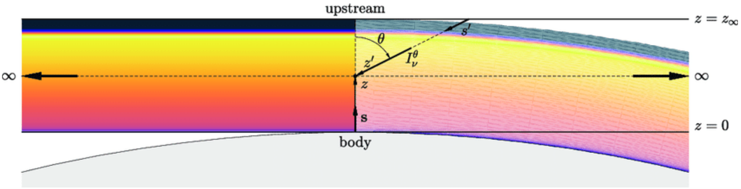
Uncoupled radiation



Coupled radiation

Shock standoff distance decreased from $\approx 1.8\mathrm{cm}$ to $\approx 4\mathrm{mm}$ after inclusion of radiation effects.

Modeling Reabsorption - Tangent Slab



Reabsorption is modeled with the standard tangent-slab formulation, with each cell's emission rescaled by the local $(T/T_{\rm ini})^4$ ratio.

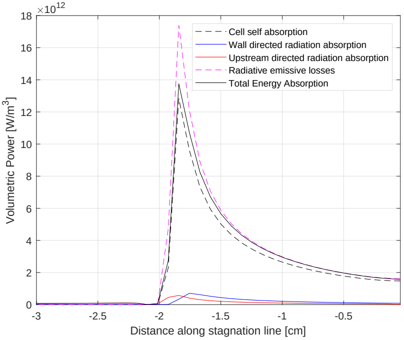
The **upstream-directed** radiation absorption term is expressed as follows:

$$\overbrace{-\nabla \vec{q}^{\,(p)}_{\rm rad}}^{\text{Absorption at cell p}} \approx -2\pi\kappa_\nu^{(p)} \left\{ \sum_{k=1}^{p-1} \frac{T_k^4}{T_{k,\text{ini}}^4} \overbrace{B_\nu^{(k)}}^{\text{Emission at cell k}} \left\{ \mathcal{E}_2\left[\tau_\nu(w^{(k)}, p)\right] - \mathcal{E}_2\left[\tau_\nu(w^{(k+1)}, p)\right] \right\} + \frac{T_p^4}{T_{p,\text{ini}}^4} \underbrace{B_\nu^{(p)}}_{\text{Emission at cell p}} \left\{ \mathcal{E}_2\left[\tau_\nu(w^{(p)}, p)\right] - \mathcal{E}_2(0) \right\} \right\} + (\ldots)$$

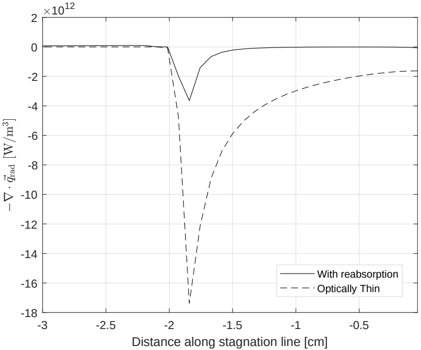
Approximate scaling comes from $\int B_\nu d\nu \propto T^4$

Modeling Reabsorption - 25km/s

The vast majority of the emitted radiation gets reabsorbed! (As expected)



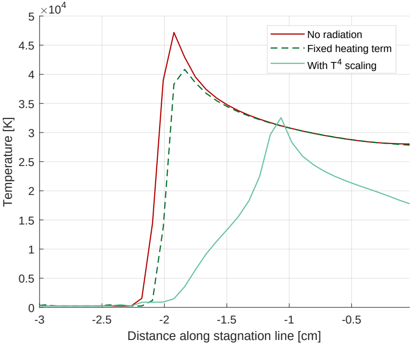
Components of $\nabla \cdot \vec{q}_{\text{rad}}$



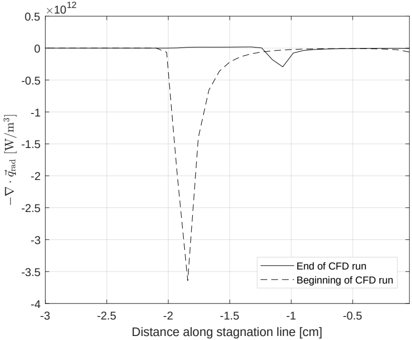
$\nabla \cdot \vec{q}_{\text{rad}}$ - Optically thin vs. w/ reabsorption

"Self absorption" dominates radiation absprption - $L_{\text{cell}} > l_{\text{optical}}$

T^4 Scaling Behavior - 25km/s



Stagnation line temperature with vs. without radiation



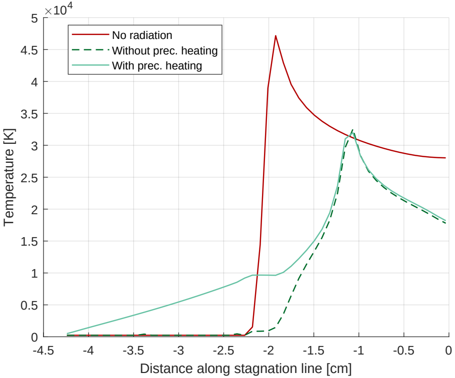
$\nabla \cdot \vec{q}_{\mathrm{rad}}$ at the start vs. end of CFD run with radiation

- T^4 scaling ensures the peak radiative sink remains attached to the shock..
- Source remains active in the region where the shock layer originally sat, pre-heating the incoming gas and smudging out the shock front.

Precursor gas heating effects

Limitations regarding the modeling of precursor heating:

- No photochemical reaction models
- The precursor gas recieves most heating from the outer shock layer. When the shock standoff decreases the precursor gas becomes far less intense
- Proper precursor modeling would require a larger mesh with the upstream boundary much more distant from the surface of the meteor



Stagnation line temperature (25km/s)

Parametric study

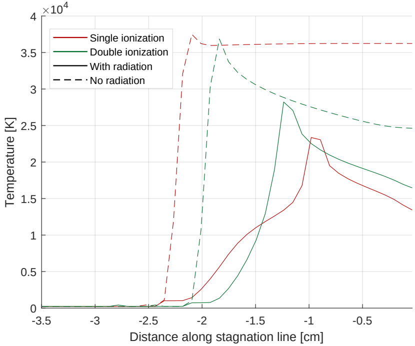
How does radiative behavior get influenced by...

- Chemical models (single vs. double ionization, 22 $\mathrm{km/s}$)
- Meteor Radius (0.3 vs. 1 m, 20 $\mathrm{km/s}$)
- Upstream velocity ($16 - 25$ $\mathrm{km/s}$)
- Thermodynamic Nonequilibrium (2T vs. 1T, 20 $\mathrm{km/s}$)

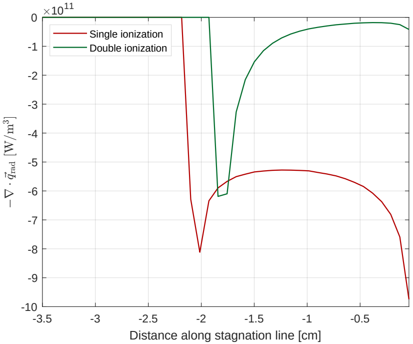
Unless stated otherwise, all case studies are performed with

- Sphere radius $R = 0.3\,\mathrm{m}$
- Double ionization enabled
- Ram pressure $0.1\,\mathrm{MPa}$
- 1 temperature (1T) model

Influence of Double Ionization - 22km/s



Stagnation line temperatures - 22km/s



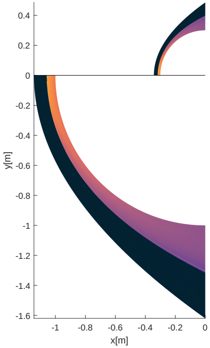
single vs. double ionization $\nabla \cdot \vec{q}_{\mathrm{rad}}$

- As expected, shock layer temperature is overestimated if double ionization is not considered
- Overestimation of radiative energy losses - double ionization should not be neglected

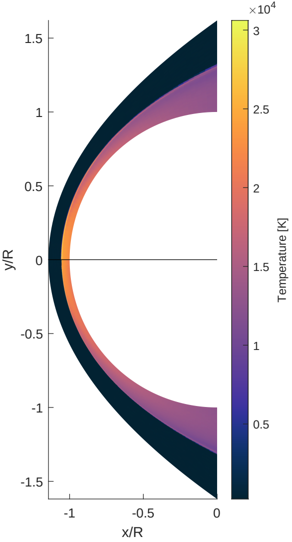
Influence of sphere Radius

Comparison between $R = 0.3\mathrm{m}$ and $R = 1\mathrm{m}$

Analysis done with upstream $v = 20\mathrm{km/s}$

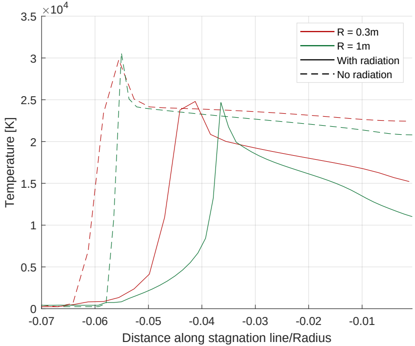


Non-scaled

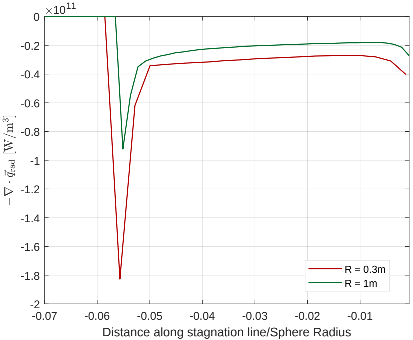


Scaled

Influence of sphere Radius - $0.3\mathrm{m}$ vs $1\mathrm{m}$



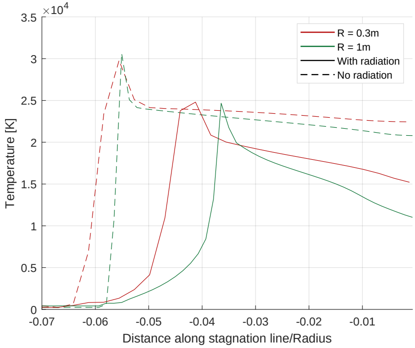
Stagnation line temperature - 20km/s



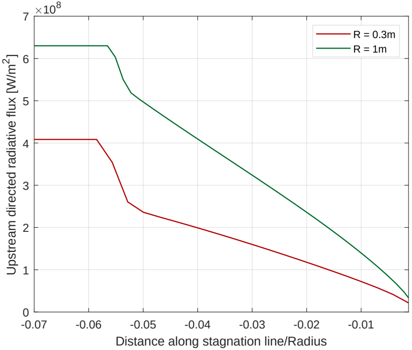
$\nabla \cdot \vec{q}_{\mathrm{rad}}$ - $R = 0.3\mathrm{m}$ vs. $R = 1\mathrm{m}$

- The $R = 1\mathrm{m}$ case loses less energy per unit volume, because a thicker shock layer is more optically thick

Influence of sphere Radius - 0.3m vs 1m



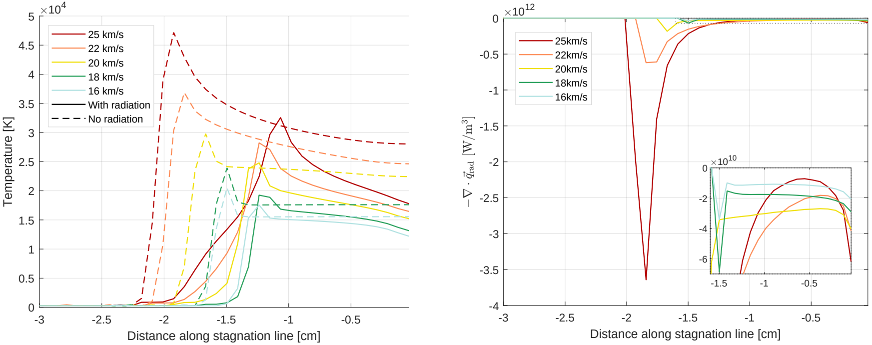
Stagnation line temperature - 20km/s



Upstream directed radiative flux

- Shock standoff retracts more in $R = 1\mathrm{m}$ because the shock layer loses $\sim 55\%$ more energy per unit area

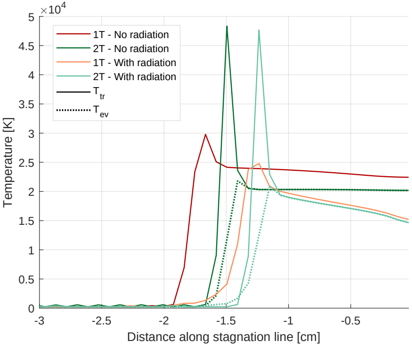
Influence of Speed



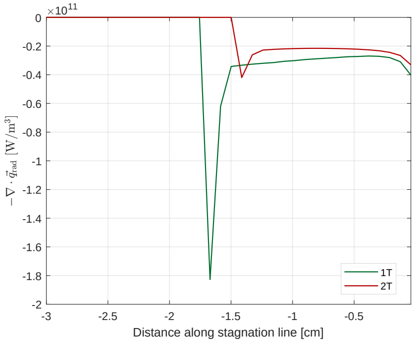
- Variation of shock standoff increases monotonically with speed
- Peak radiative losses increase monotonically with speed
- Beyond 20km/s, large optical thickness results in reduced losses in inner shock layer.

Influence of non-LTE Phenomena (2T model)

Behavior in the shock layer is similar (when $T_{\mathrm{ev}} \approx T_{\mathrm{tr}}$)



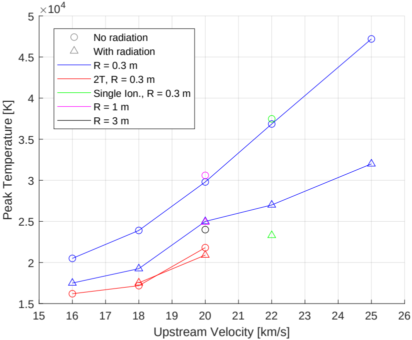
Stagnation line temperatures - 20km/s

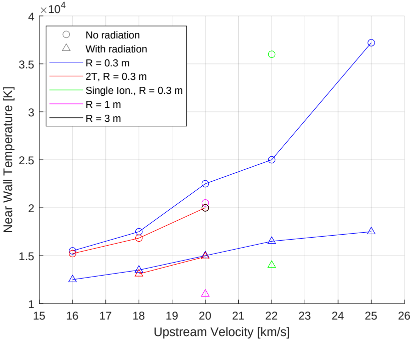


1T vs 2T $\nabla \cdot \vec{q}_{\mathrm{rad}}$ - 20km/s

- 1T: Instant excitation of internal d.o.f's as flow crosses shock
- 2T: Gradiual excitation of internal d.o.f's as T_{ev} increases
- Result: sharper emissions peak in 1T model

Asymptotic Trends - Peak Temperature





- As velocity increases, the gap between the peak temperatures with/without radiation widens
- Up to 25km/s it is not yet possible to see a plateau in peak temperature

Conclusions

Coupling and reabsorption

- The optically thin assumption is strongly nonphysical
- The $(T)^4$ rescaling helps the radiative source term evolve with the flowfield, stabilizing CFD compared to a frozen radiative source

Work in progress

- Comparison with results from the literature is still in progress - crucial to assess feasibility of approximations
- Only one "loop" of weakly coupled cycle was done so far. Further iterating might increase accuracy.

Annex - Simulation Matrix

Radius (m)	Velocity (km/s)	Energy reservoirs	Number of species	Other conditions	Objective	Status
0.3	25	1	13	Ram pressure $= 0.1\,\text{MPa}$	Assess optically thin model	✓
0.3	16 18 20 22	1	13	Ram pressure $= 0.1\,\text{MPa}$	Assess influence of velocity	✓ ✓ ✓ ✓
0.3	22	1	11	Ram pressure $= 0.1\,\text{MPa}$	Assess influence of chemical model	✓
1 3	20	1	13	Ram pressure $= 0.1\,\text{MPa}$	Assess influence of sphere radius	✓ IP
0.3	16 18 20 22	2	13	Ram pressure $= 0.1\,\text{MPa}$	Assess influence of 2T model	IP ✓ ✓ DNC
0.3	25	1	13	Ram pressure $= 0.1\,\text{MPa}$	Assess influence of precursor heating	DNC
0.2	16 22	2	13	Density$= 5 \times 10^{-4}\,\text{kg/m}^3$	Comparison with Johnston (2016)	IP DNC

IP = In progress DNC = Did not converge

Tangent Slab Discretization

In the tangent slab model, the divergence of heat flux is given by:

$$- \left(\nabla \cdot \boldsymbol{q}_{\mathrm{rad}} \right)_{\nu} = -4\pi \varepsilon_{\nu}(z) - 2\pi \kappa_{\nu} \left\{ \int_0^z j_{\nu}(s) \, \frac{\mathrm{d}\mathcal{E}_2 \left[\tau_{\nu}(s,z) \right]}{\mathrm{d}\tau_{\nu}} \, \mathrm{d}s \right.$$

$$\left. + \int_z^{z\infty} j_{\nu}(s) \, \frac{\mathrm{d}\mathcal{E}_2 \left[\tau_{\nu}(z,s) \right]}{\mathrm{d}\tau_{\nu}} \, \mathrm{d}s - I^{+}_{\nu,b} \, \mathcal{E}_2 \left(\tau_{\nu}(0,z) \right) \right\}.$$

By definition of the source function S_{ν} and optical thickness τ_{ν}: $\varepsilon_{\nu} \equiv \kappa_{\nu} S_{\nu}$, and $\mathrm{d}\tau_{\nu}(s,z) \equiv -\kappa_{\nu}(s) \, \mathrm{d}s$ (the optical distance to z *decreases* as s increases)

$$\int_0^z j_{\nu}(s) \, \frac{\mathrm{d}\mathcal{E}_2 \left[\tau_{\nu}(s,z) \right]}{\mathrm{d}\tau_{\nu}} \, \mathrm{d}s = - \int_{\tau_{\nu}(0,z)}^{0} S_{\nu}(s) \frac{\mathrm{d}\mathcal{E}_2}{\mathrm{d}\tau_{\nu}} \, \mathrm{d}\tau_{\nu} \, .$$

Splitting the integral into sections covering one cell length, and assuming that S_{ν} is constant within a given cell collapses the integral into a sum of integrals of exact differentials:

$$\int_0^z j_{\nu}(s) \, \frac{\mathrm{d}\mathcal{E}_2 \left[\tau_{\nu}(s,z) \right]}{\mathrm{d}\tau_{\nu}} \, \mathrm{d}s = - \sum_{k=1}^{p-1} S_{\nu}(s^{(k)}) \int_{\tau(s^{(k-)},z)}^{\tau(s^{(k^{+})},z)} \frac{\mathrm{d}\mathcal{E}_2}{\mathrm{d}\tau_{\nu}} \, d\tau_{\nu} - S_{\nu}(z) \int_{\tau(s^{(p-)},s^{(p)})}^{0} \frac{\mathrm{d}\mathcal{E}_2}{\mathrm{d}\tau_{\nu}} \, d\tau_{\nu}$$

Note: here, p represents the index of the cell with radial coordinate z (where the source term is being calculated), and k represents the index of other cells. $+$ and - serve to indicate the cell walls closest and farthest away (respectively) from the upstream boundary.

Tangent Slab Discretization

Evaluating each integral analytically yields a series of $\mathcal{E}_2$ evaluated at the cell faces. The upstream directed integral can be written:

$$\int_0^z j_\nu(s)\,\frac{\mathrm{d}\mathcal{E}_2}{\mathrm{d}\tau_\nu}\,\mathrm{ds} \approx \sum_{k=1}^{p-1} S_\nu^{(k)}\left\{\mathcal{E}_2\left[\tau_\nu(k^-,z)\right] - \mathcal{E}_2\left[\tau_\nu(k^+,z)\right]\right\} + S_\nu^{(p)}\left\{\mathcal{E}_2\left[\tau_\nu(p^-,z)\right] - \mathcal{E}_2(0)\right\}$$

By Kirchoff's law, the source function is equal to the Planck spectrum $S_\nu = B_\nu$, whose integral scales as T^4. As such, this temperature dependence is introduced in the formulation.

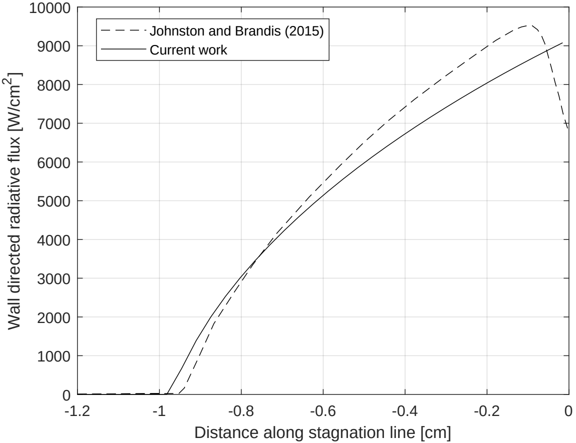
$$\int_0^z j_\nu(s)\,\frac{\mathrm{d}\mathcal{E}_2}{\mathrm{d}\tau_\nu}\,\mathrm{ds} \approx \sum_{k=1}^{p-1} \frac{T_k^4}{T_{k,\text{initial}}^4} S_\nu^{(k)}\left\{\mathcal{E}_2\left[\tau_\nu(k^-,z)\right] - \mathcal{E}_2\left[\tau_\nu(k^+,z)\right]\right\} + \frac{T_p^4}{T_{p,\text{initial}}^4} S_\nu^{(p)}\left\{\mathcal{E}_2\left[\tau_\nu(p^-,z)\right] - \mathcal{E}_2(0)\right\}$$

Note that although it is true that $\int S_\nu d\nu$ scales exactly as T^4 under LTE, the same relation is not true spectrally. As such, this scaling serves as an approximation.

The wall directed radiation integral is discretized in the same manner.

Tangent Slab Discretization Validation

Comparison with $R = 0.2\mathrm{m}$, $v = 16\mathrm{km/s}$, $\rho = 5 \cdot 10^{-4}\mathrm{kg/m^3}$.

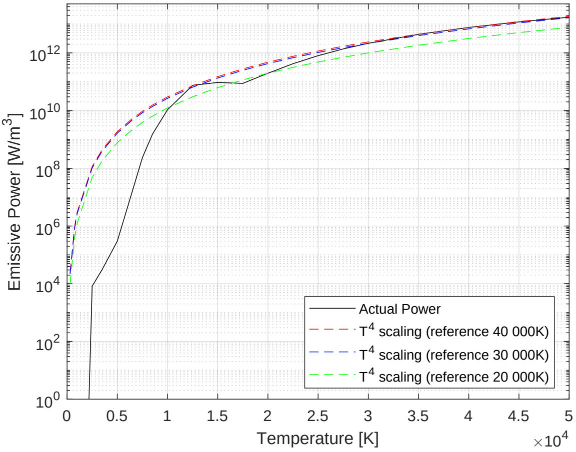


Difference near wall due to isothermal wall (Johnston, 2015) vs. adiabatic wall (current work).

C. O. Johnston and A. M. Brandis, "Aerothermodynamic Characteristics of 16–22 km/s Earth Entry," AIAA 2015-3110, 2015.

Emissive ST scaling

T^4 scaling approximation for emissive source is more accurate at higher temperatures.



At lower temperatures emissions are overestimated