Business-as-Usual and Orbital Debris Path

Anelí Bongers and José L. Torres (UMA)

June 3-5, 2026 Milano

Outline

1. Introduction (Motivation)
2. Business-as-Usual and Environmental Externalities
3. The Dynamic General Equilibrium Integrated Space-Economy model
4. Competitive Decentralized Equilibrium
5. Decentralized Solution Approach
6. Data and Calibration
7. Results: Simulations
8. Concluding remarks

1. Introduction

- Although human exploration and economic exploitation of the outer space is quite recent (the first successful launch of a human-made spacecraft occurred in 1957), a number of market failures are arising at rocket speed as commercial, military, and scientific activities in space are continuously expanding.
- Outer space is an example of an extraterrestrial common resource and hence is subject to comparable economic failures to other international commons on the Earth (i.e., fisheries in international waters, the atmosphere, or Antarctica).
- Negative externality: Space pollution (Orbital debris).
- Debris is a type of space pollution that could have dramatic consequences for commercial and other activities in outer space (Kessler and Cour-Palais, 1978).

2. Business-as-Usual and Environmental Externalities

- Computing the BaU scenario, i.e., solving for a decentralized equilibrium in IAMs with environmental externalities, is not a trivial task. As noted by Shiell and Lyssenko (2008), simple optimization procedures tend to internalize the cost of pollution, which contradicts the very notion of BaU.
- Krusell and Smith (2024) propose a fixed-point iteration method to compute the competitive equilibrium in the context of climate change.
- The decentralized equilibrium is obtained by iterating on the debris stock trajectory while repeatedly solving a nonlinear programming problem in which agents take debris-related damages as exogenous.

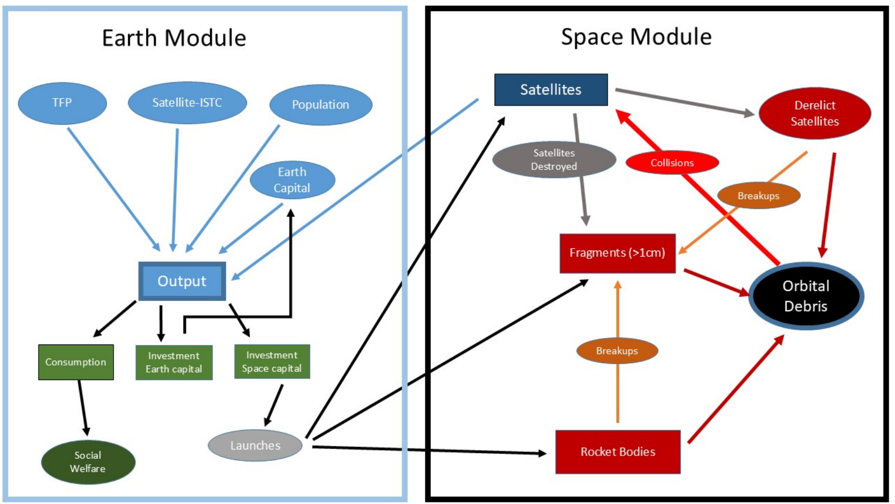
2. Business-as-Usual and Environmental Externalities

- In BaU, there are no corrective policy instruments, such as Pigouvian taxes, tradable permits, or regulatory constraints, that would align private incentives with social welfare. Prices therefore fail to reflect the true social cost of externality-generating activities.
- The BaU scenario typically serves as a benchmark against which policy scenarios are evaluated. Comparing welfare, environmental outcomes, and economic trajectories under BaU and policy regimes allows researchers to quantify the magnitude of the externality, assess the efficiency gains from internalization, and evaluate the effectiveness of alternative policy instruments.

3. The model

- We develop a general equilibrium IAM (Integrated Assessment Model), similar to the models used in the climate-change literature (Nordhaus, 1992).
- The economy part of the model is an optimal neoclassical growth model (A Ramsey-Cass-Koopmans model). Ramsey (1928), Cass (1965), Koopmans (1965).
- The physical part of the model consists in a simplified orbital debris evolutionary model (a system of three in-difference equations).

3. The model: The economy



3. The model: The economy

- The economy part of the model is based on the optimal neoclassical growth model tailored to consider human activity in space.
- Time is discrete running to an infinity horizon.
- The economy (the world) is inhabited by a large number of identical households.
- There is a representative household with instantaneous utility $U(\hat{c}_t, N_t)$, defined over per capita consumption $\hat{c}_t$, and where N_t is population.
- The aggregate consumption, c_t, is defined as $c_t = \hat{c}_t N_t$.
- The function $U(\hat{c}_t, N_t)$ is the flow of utility which it is assumed to represent social well-being.

3. The model: The economy

- The problem to be solved by the stand-in household consists in maximizing the sum of discounted utility.

$$max_{\{\hat{c}_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t U(\hat{c}_t, N_t) \tag{1}$$

where $\rho > 0$ is the subjective intertemporal preference parameter or the pure rate of social time preference. The discount factor, $0 < \beta < 1$, is defined as $\beta = 1/(1+\rho)$.

3. The model: The Economy

- This household satisfies the following budget constraint which coincides with the feasibility constraint in the final-good sector:

$$c_t + i_t + h_t = r_t^k k_t + r_t^s s_t + w_t N_t + \pi_t \tag{2}$$

  where i_t is investment in physical capital other than satellites (Earth's capital), h_t is investment in satellites (Space capital), with includes all costs to insert satellites into orbit, the rental rate of Earth capital is r_t^k, the rental rate of satellites is r_t^s, and w_t is labor income. They also receive dividends, π_t.
- All prices are defined in output units and normalized to one, but the price (user cost) of Space capital differs from the price of Earth capital.

3. DISE-2024: The economy

- Output is assumed to be a function of aggregate productivity, the stock of physical capital in the Earth, k_t, the stock of satellites, s_t, and labor, N_t,

$$y_t = a_t f(k_t, s_t, N_t) \tag{3}$$

where a_t is the total factor productivity (TFP), representing Hicks-neutral technological change.

- Labor is assumed to be equal (or proportional) to population, and hence, labor growth rate equals to population growth rate.

3. DISE-2024: The economy

- The capital other than satellites accumulation process is the standard inventory equation:

$$k_{t+1} = (1 - \delta_k)k_t + i_t \tag{4}$$

where $0 < \delta_k < 1$ is the capital depreciation rate.

- The stock of satellites, measured in final output units as an equipment asset, is denoted by s_t, and is given by the following process,

$$s_{t+1} = (1 - \delta_s)s_t + q_t h_t - x_t \tag{5}$$

where $0 < \delta_s < 1$ is the depreciation rate for satellites, and x_t is the loss of satellites assets by collisions (damage from pollution).

3. DISE-2024: The economy

- Exogenous growth sources: Neutral technological progress, investment-specific technological change for satellites, and labor growth.
- Neutral technological progress is characterized as,

$$a_{t+1} = exp(g_{a,t})a_t \tag{6}$$

where $g_{a,t}$ is the growth rate of TFP defined as,

$$g_{a,t} = g_{a,0}exp(-\delta_a t) \tag{7}$$

where δ_a is the decay rate in the TFP growth rate.

- We assume a similar specification for the satellite investment-specific technological progress,

$$q_{t+1} = exp(g_{q,t})q_t \tag{8}$$

where $g_{q,t}$ is the growth rate of ISTC defined as,

$$g_{q,t} = g_{q,0}exp(-\delta_q t) \tag{9}$$

where δ_q is the decay rate in ISTC for satellites.

3. DISE-2024 (The economy)

- Finally, we define the dynamics of the population, N_t. Population is another source of growth as it is assumed that labor equals population. Population dynamics is defined as,

$$N_{t+1} = N_t \left(\frac{N^*}{N_t} \right)^{\varsigma} \tag{10}$$

where ς is the population growth rate, and N^* is the asymptotic population.

- UN estimates: World population maximum at 10,289,515 thousands in the year 2084 (10,186,608 thousands for 2100).

3. DISE-2024 (The space: The stock of orbital debris)

- The stock of debris, as a function of their size, $D_{i,t}$, can be defined as:

$$D_{i,t} = W_t + Z_t + F_{i,t} \quad \mathit{for} \quad i = 1, 2, 3 \tag{11}$$

  where we distinguish three types of debris: W_t is the stock of derelict satellites abandoned in orbit, Z_t is the number of last stages rocket bodies, and $F_{i,t}$ are fragments larger than 10 cm ($i = 1$), fragments larger than 1 cm ($i = 2$), and fragments larger than 1 mm ($i = 3$).
- The key distinction is that fragments are dead debris, but both derelict satellites and rocket bodies can breakup, generating a large number of fragments.

3. DISE-2024 (The space: Derelict satellites)

- A source of debris are derelict satellites, W_t. Derelict satellites are end-live satellites not operational anymore (generally due to run out of fuel) that are not de-orbited and instead they are abandoned in orbit. In each period, the number of end-live satellites is $\delta_s S_t$. Each period, a fraction χ of end-live satellites are abandoned in orbit. Hence, the law of motion of derelict satellites is given by,

$$W_{t+1} = (1 - \delta_w - \varepsilon_w) W_t - \theta(D_{2,t} + (1 - v_t) S_t) W_t + \chi \delta_s S_t \quad (12)$$

where δ_w is the natural decay rate of debris and ε_w is the fraction of derelict satellites that explode each period. In case of explosion, it is assumed that the number of debris fragments (larger than 10 cm) is $\phi_w \varepsilon_w W_t$. The number of debris fragments produced in case of collision of derelict satellites with any other type of debris is $\gamma_w \theta D_{2,t} W_t$.

3. DISE-2024 (The space: Rocket bodies)

- The accumulation process for rocket bodies is similar. Therefore, the stock of (upper stages) body rockets can be defined as

$$Z_{t+1} = (1 - \delta_z - \varepsilon_z)Z_t - \theta(D_{2,t} + (1 - v_t)S_t)Z_t + \varphi L_t \tag{13}$$

  where δ_z is the natural decay rate of rocket bodies, and ε_z is the fraction of body rockets that breakup each period. The number of debris fragments (larger than 10 cm) produced by the breakup of rocket bodies is $\phi_z \varepsilon_z Z_t$. The number of debris fragments produced by the collision of rocket bodies with any other type of debris is $\gamma_z \theta D_{2,t} Z_t$.

3. DISE-2024 (The space: Fragments)

- Finally, the law of motion of debris fragments (larger than 10 cm) is given by,

$$F_{1,t+1} = (1-\delta_f)F_{1,t} + \omega L_t + \gamma_s X_t + \phi_w \varepsilon_w W_t + \phi_z \varepsilon_z Z_t + \gamma_w \theta D_{2,t} W_t + \gamma_z \theta D_{2,t} Z_t \quad (14)$$

where δ_f is the natural decay rate of debris fragments, γ_s is the amount of debris creates by a collision and destruction of operational satellites, and ω is the amount of debris produces per launch.

- The stock of orbital debris for computing damages is:

$$D_{2,t} = W_t + Z_t + F_{2,t} \tag{15}$$

where

$$F_{2,t} = (1 + \Gamma)F_{1,t} \tag{16}$$

where Γ is the proportion of fragments between 1 cm and 10 cm to fragments larger than 10 cm.

3. DISE-2024 (The space: Damages)

- The number of destroyed satellites every period by collision with debris is assumed to be a function of debris and operational satellites,

$$X_t = (1 - v_t)\theta D_{2,t} S_t \tag{17}$$

  where $\theta > 0$ is a parameter representing the probability of collision, $0 \leq v_t \leq 1$ represents successful avoidance collision maneuvers, and D_t is the number of orbital debris.
- When $(1 - v_t)\theta D_{2,t} = 1$, any space assets are destroyed rendering the space unusable. This point is reached when the pollution stock is $D_{2,t} = 1/((1 - v_t)\theta)$.

4. Competitive Decentralized Equilibrium

- The household maximization problem, taking into account the accumulation process for both capital and satellites, is given by,

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t U(\hat{c}_t) N_t$$
$$- \sum_{t=0}^{\infty} \lambda_t [c_t + k_{t+1} - (1-\delta_k) k_t$$
$$+ \frac{1}{q_t(1-m_t)} [s_{t+1} - (1-\delta_s) s_t + (1-v_t)\theta \overline{D}_{2,t} s_t]$$
$$- r_t^k k_t - r_t^s s_t - w_t N_t - \pi_t] \tag{18}$$

where $\overline{D}_{2,t}$ is the exogenous amount of orbital debris. Hence, orbital debris affects the optimal decision by the agent but their social costs are not internalized.

- First order conditions from $t = 0, 1, ..., \infty$, are:

$$\frac{\partial \mathcal{L}}{\partial c_t} = \beta^t U_c'(c_t) - \lambda_t = 0 \tag{19}$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = -\lambda_t + \lambda_{t+1}(1 - \delta_k + r_{t+1}^k) \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial s_{t+1}} = \frac{\lambda_t}{q_t(1-m_t)}$$
$$-\frac{\lambda_{t+1}}{q_{t+1}(1-m_{t+1})}[1 - \delta_s - (1-v_t)\theta\overline{D}_{2,t+1}] + \lambda_{t+1} r_{+1}^s = 0 \tag{21}$$

4. Competitive Decentralized Equilibrium

- From the maximization problem, solving for the Lagrangian's multiplier, equilibrium conditions are given by,

$$U_c'(c_t) = \beta E_t U_c'(c_{t+1})[1 - \delta_k + r_{t+1}^k] \tag{22}$$

$$U_C'(c_t) = q_t(1 - m_t)\beta E_t U_c'(c_{t+1}) \left[\frac{1 - \delta_s - (1 - v_t)\theta \overline{D}_{2,t+1}}{q_{t+1}(1 - m_{t+1})} + r_{t+1}^s \right] \tag{23}$$

4. Competitive Decentralized Equilibrium

- From the profit maximization problem of the representative firm we have:

$$r_t^k = a_t f_k'(k_t, s_t, N_t) \tag{24}$$

$$r_t^s = a_t f_s'(k_t, s_t, N_t) \tag{25}$$

$$w_t = a_t f_N'(k_t, s_t, N_t) \tag{26}$$

- Equilibrium conditions are obtained by substituting (24) and (25) into (22) and (23),

$$U'_c(c_t) = \beta E_t U'_c(c_{t+1})[1 - \delta_k + a_{t+1} f'_k(k_{t+1}, s_{t+1})] \tag{27}$$

$$U'_c(c_t) = q_t(1 - m_t)\beta E_t U'_c(c_{t+1}) \left[\frac{1 - \delta_s - (1 - v_t)\theta \overline{D}_{2,t+1}}{q_{t+1}(1 - m_{t+1})} + a_{t+1} f'_s(k_{t+1}, s_{t+1}) \right] \tag{28}$$

4. Competitive Decentralized Equilibrium

- Equating both expressions we find that the arbitrage condition for investing in both Earth and space capital is,

$$\left[1-\delta_k + a_{t+1} f_k'(k_{t+1}, s_{t+1})\right] = q_t(1-m_t)\left[\frac{1-\delta_s-(1-v_t)\theta\overline{D}_{2,t+1}}{q_{t+1}(1-m_{t+1})} + a_{t+1} f_s'(k_{t+1}, s_{t+1})\right] \quad (29)$$

5. Functional forms, Initial values, and Calibration (I)

- CRRA Utility function,

$$U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \tag{30}$$

- Cobb-Douglas aggregate production function,

$$y_t = a_t k_t^{\alpha_1} s_t^{\alpha_2} N_t^{1-\alpha_1-\alpha_2} \tag{31}$$

5. Functional forms, Initial values, and Calibration (II)

Initial values (year 2023)

Variable	Value	Units	Sources
World GDP	184.65	Trillions int. US$	World Bank
Earth capital stock	554.23	Trillions int. US$	IMF and BEA
Space Capital stock	1.72	Trillions int. US$	IMF and BEA
World population	8,056	Millions inhabitants	UN
Operational satellites	8,500	Units	ESA
Number of launches	217	Units	NASA
Derelict satellites	3,524	Units	Model
Rocket bodies	2,050	Units	NASA
Orbital debris $> 10cm$	36,500	Units	NASA/ESA
Orbital debris $> 1cm$	1,036,500	Units	NASA/ESA

5. Functional forms, Initial values, and Calibration (III)

Calibration of the economic parameters of the model

Parameter	Definition	Estimate
ρ	Rate of social time preference	0.015
σ	Risk aversion parameter	1.5
α_1	Earth capital elasticity	0.3479
α_2	Space capital elasticity	0.0021
δ_k	Earth capital depreciation rate	0.07
δ_s	Satellite depreciation rate	0.15
ς	Population growth rate	0.05
g_a	TFP growth rate	0.015
g_q	Satellites ISTC growth rate	0.03
δ_a	Decay rate in TFP growth rate	0.001
δ_q	Decay rate in ISTC growth rate	0.005
μ	Conversion parameter	4,941

5. Functional forms, Initial values, and Calibration (IV)

Calibration of the physical parameters of the model

Parameter	Definition	Estimate
δ_f	Fragment natural decay rate	0.01
δ_w	Derelict satellites natural decay rate	0.00015
δ_z	Rocket bodies natural decay rate	0.00015
θ	Risk of collision	1.25×10^{-10}
ω	Debris $> 10cm$ per launch	4.00
γ_s	Debris $> 10cm$ per satellite collision	70
γ_w	Debris $> 10cm$ per derelict satellite collision	70
γ_z	Debris $> 10cm$ per rocket collision	70

5. Functional forms, Initial values, and Calibration (V)

Calibration of the physical parameters of the model

Parameter	Definition	Estimate
χ	Fraction of derelict satellites	0.40
ϕ_w	Fragments $> 10cm$ per satellite breakup	44.6
ϕ_z	Fragments $> 10cm$ per rocket body breakup	100.2
ε_w	Fraction of dead satellites breakups	0.001
ε_z	Fraction of rocket bodies breakups	0.0012
φ	Rocket bodies per launch	0.60
η	Satellites per launch	13.60
Γ	Debris$> 1cm$ to $> 10cm$	34.3

6. Decentralized solution approach

- We adopt a conceptually similar approach to that of Krusell and Smith (2024) but adapt it to the context of orbital environmental externalities, using a nonlinear programming (NLP) formulation instead of dynamic programming.
- Rather than computing policy functions, we solve a finite-horizon dynamic optimization problem directly at each iteration using numerical optimization tools.
- The model solution represents a decentralized economy in which agents maximize utility and profits based on private returns, taking the trajectory of orbital debris and associated damages as exogenous.

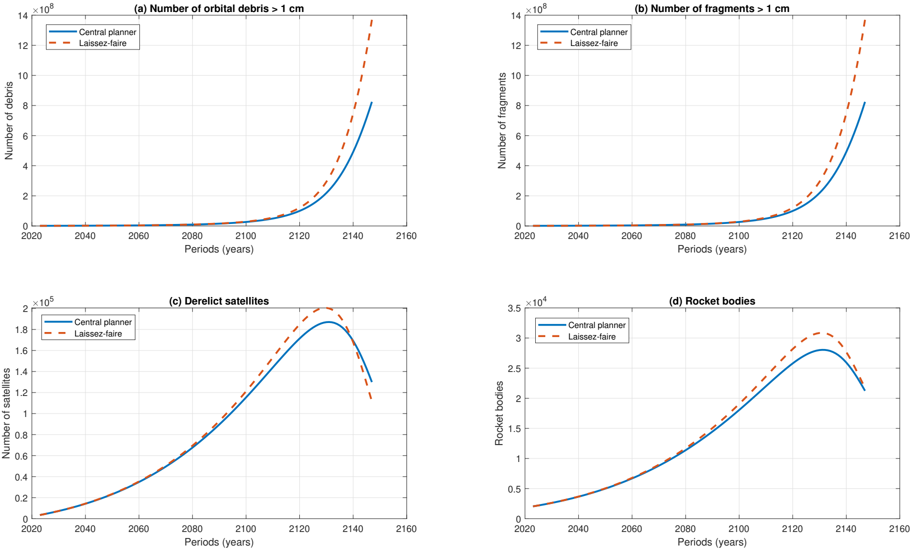
6. Decentralized solution approach

- The fixed-point pseudo-algorithm is given is the following:
- **Initialize:** Set initial conditions and guess for debris paths $(W(t), Z(t), F(t)$, etc.)
- **Main Iteration Loop:** iter $=$ 1 to max_iter
  - Solve the NLP optimization problem: maximize total discounted utility
  - Update orbital debris dynamics
  - Propagate state to $t+1$
  - Update damage (satellites destroyed by collisions)
  - Compute difference between updated and previous satellites destroyed X_t
  - If converged (difference $<$ tolerance) **Break**
- **Output:** Optimal path for economic variables and orbital debris for a decentralized setting.

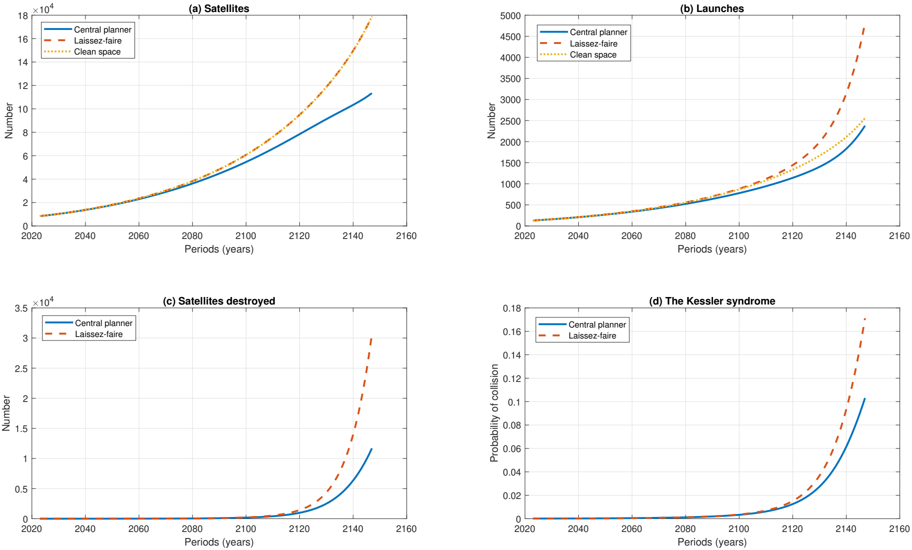
6. Decentralized solution approach

- Horizon: 200 years. Initial year 2023.
- Solution of the optimal growth model as a nonlinear programming problem.
- The model has 5629 variables and 5525 constraints.
- Code in GAMS (General Algebraic Modeling System) using the algorithm CONOPT. Cai (2019): GAMS is more flexible than MATLAB and CONOPT is more reliable and efficient than *fmicon* in solving NLP problems.
- Alternative solvers for MATLAB are KNITRO and CasADi.

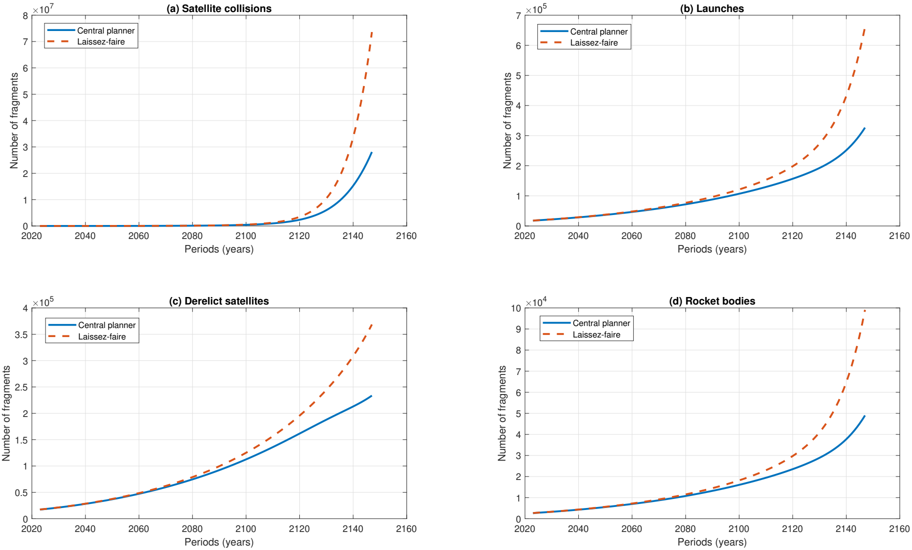
7. Simulation results



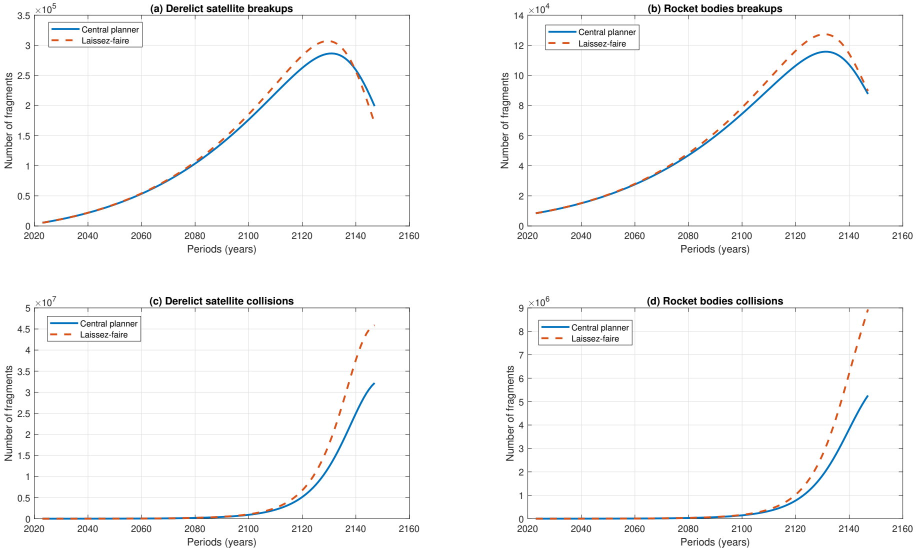
7. Simulation results



7. Simulation results



7. Simulation results



7. Conclusions

- Space is an international common, and thus subject to "the tragedy of commons". Orbital debris is one consequence.
- The difference between the BaU paths and the first-best central planner solution highlights the magnitude of inefficiencies generated by unregulated space activity.
- Kessler syndrome accelerates in the BaU scenario, although also appears in the fist-best world.
- Traditional mitigation policy instruments are not enough to avoid the Kessler syndrome.

Thank you very much for your attention