



THE CRITICALITY OF ORBITAL REGIONS: FROM LOCAL EFFECTS TO THE GLOBAL HEALTH OF THE SPACE ENVIRONMENT

Milan-
Workshop

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Agenda



From Space
Environment
to Network
Theory



The Criticality
of Orbital
Regions



From Orbital
Regions to
Environment
and Back



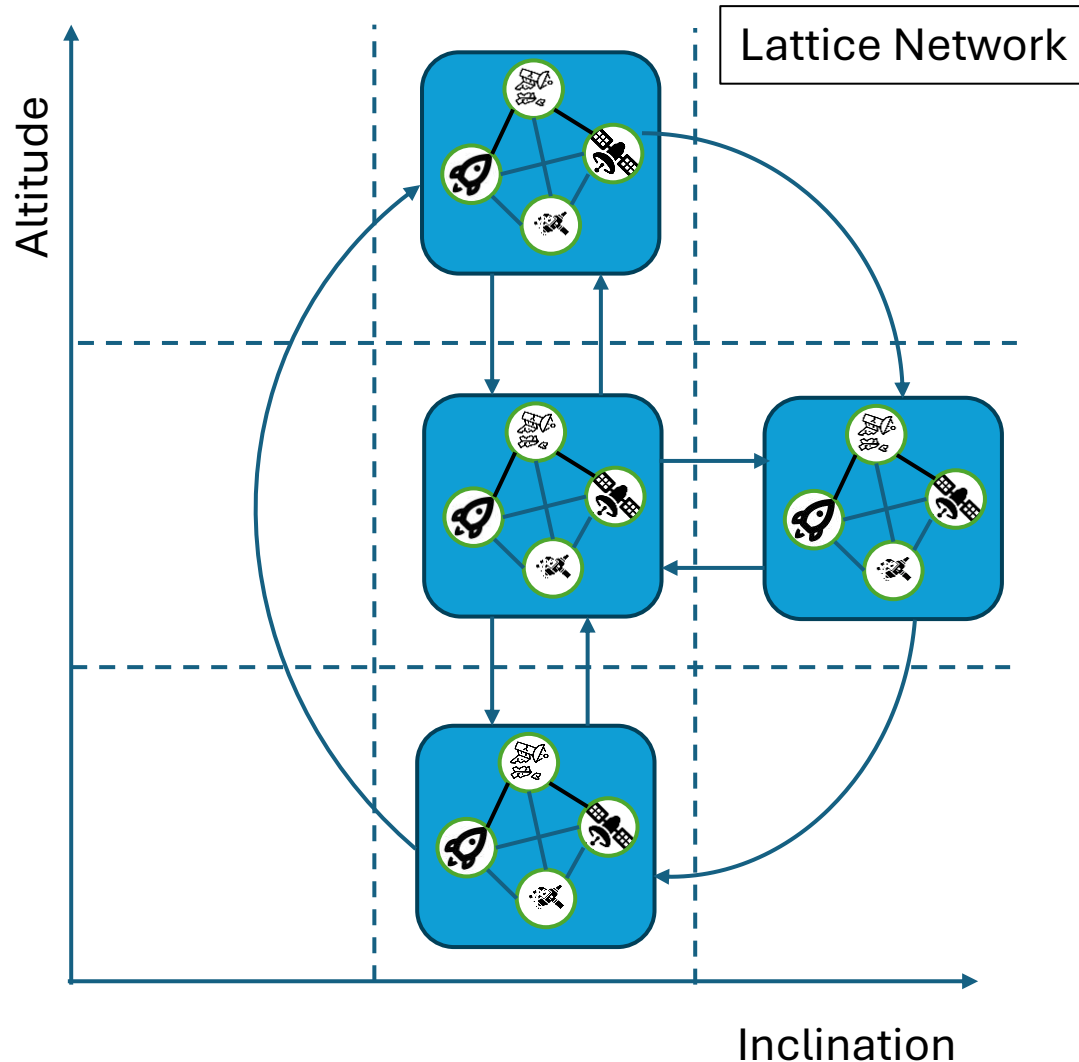
Project PERSEO: A Percolation Theory of the Space Environment

Agenda



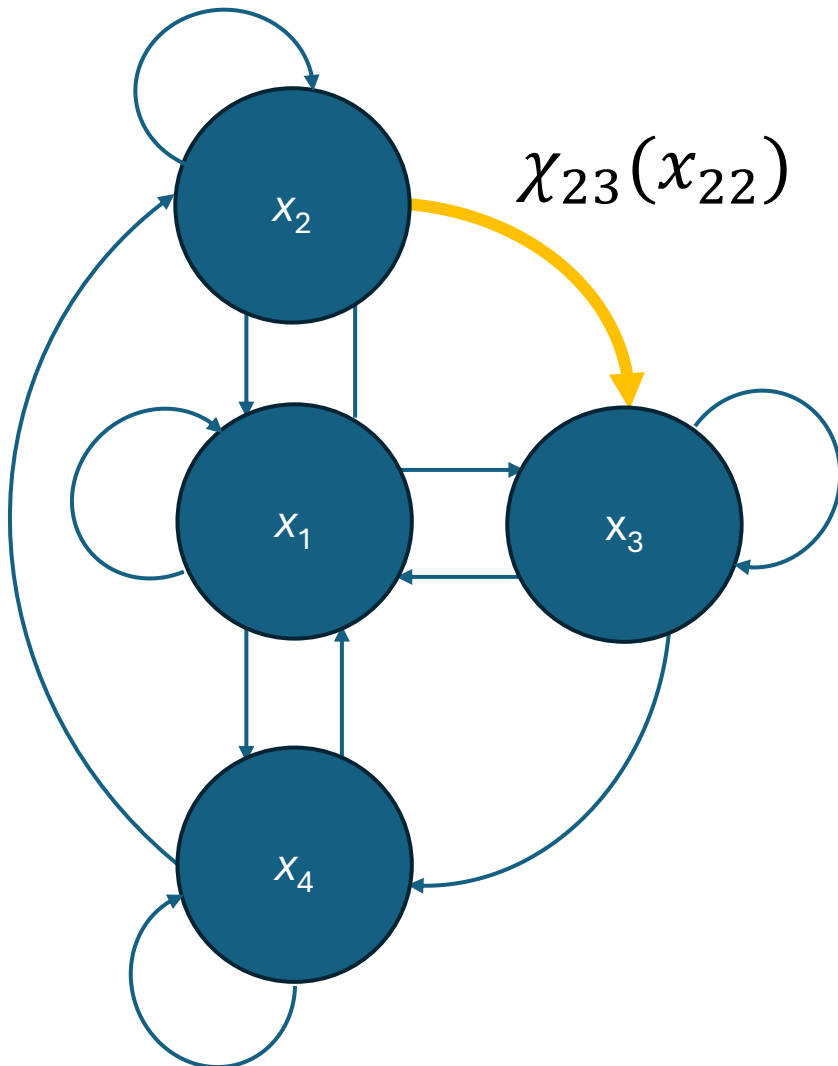
From Space
Environment
to Network
Theory

From Physical Space to Network Abstraction



- Orbital regions are defined by sets of orbital parameters.
- Each region is occupied by different species of objects.
- Each species is connected to other species and each region is connected to all other regions.

From Physical Space to Network Abstraction



- The number of objects per species in each node depends on three factors:
 - Natural dynamics (e.g. orbital decay)
 - Random events (e.g. collisions)
 - New launches
- **Dynamics driven by events with rates χ :**

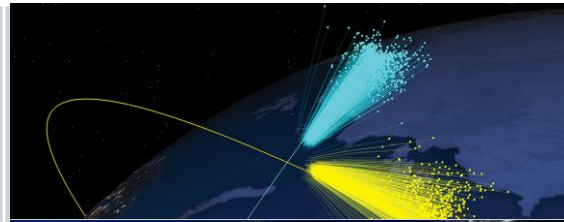
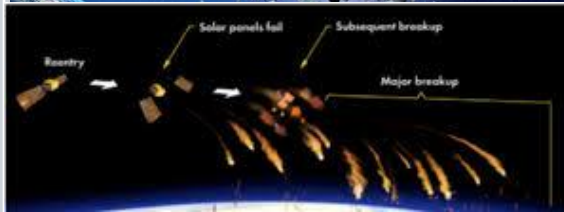
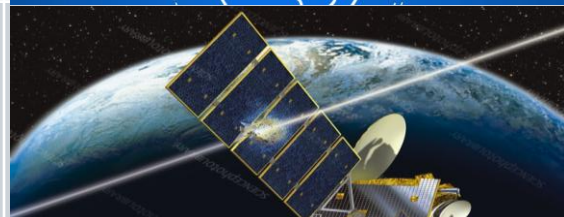
$$\mathbf{X}_{k+1} = \mathbf{X}_k + f(\chi(\mathbf{X}_k)) + \Delta(\chi(\mathbf{X}_k)) + \Lambda(\chi(t_k))$$

Natural dynamics

Jump processes

New launches

Event Rates

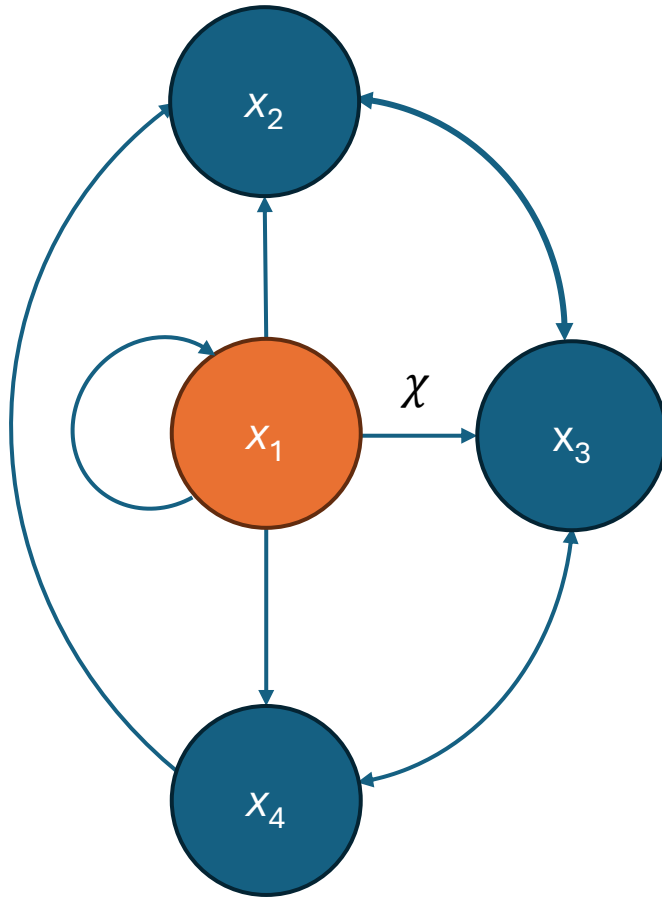
Type of event	Event	Event rate	
Collisions	Fragment generation, removal of colliding objects	χ^C	
Flow of fragments	Flow of fragments	χ^F	
Failed PMD	Flow from active payload to defunct	χ^{PMD}	
Orbit decay	Flow from upper to lower shells	χ^D	
Satellite failure	Flow from active payload to defunct	χ^{SC}	

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The Criticality
of Orbital
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The Criticality of Orbital Regions



- The criticality of region 1 is how much region 1 affects the other regions and itself.
- We consider all direct and indirect links from region 1 to all other regions
- Each link defined by multiple event rates and their severity.
- **Example, take the flow of fragments $\chi^{\mathcal{F}}$ from 1 to 3.**
- The severity of that event, is defined as:

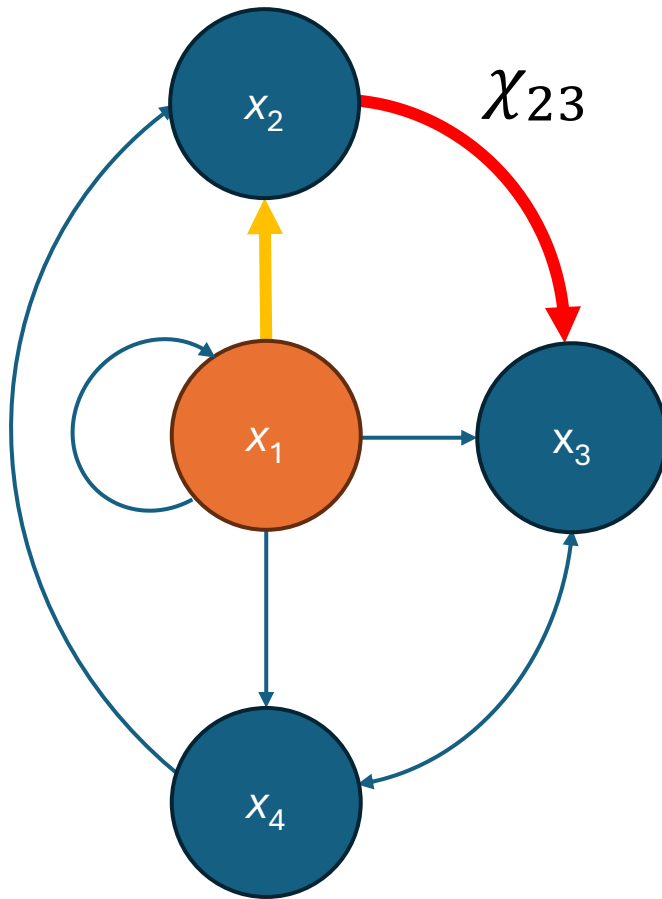
$$s = \frac{\Delta\tau}{\Delta\tau + a}$$

Expected increase in collision rate τ in region 3

Expected resident time of objects in region 3

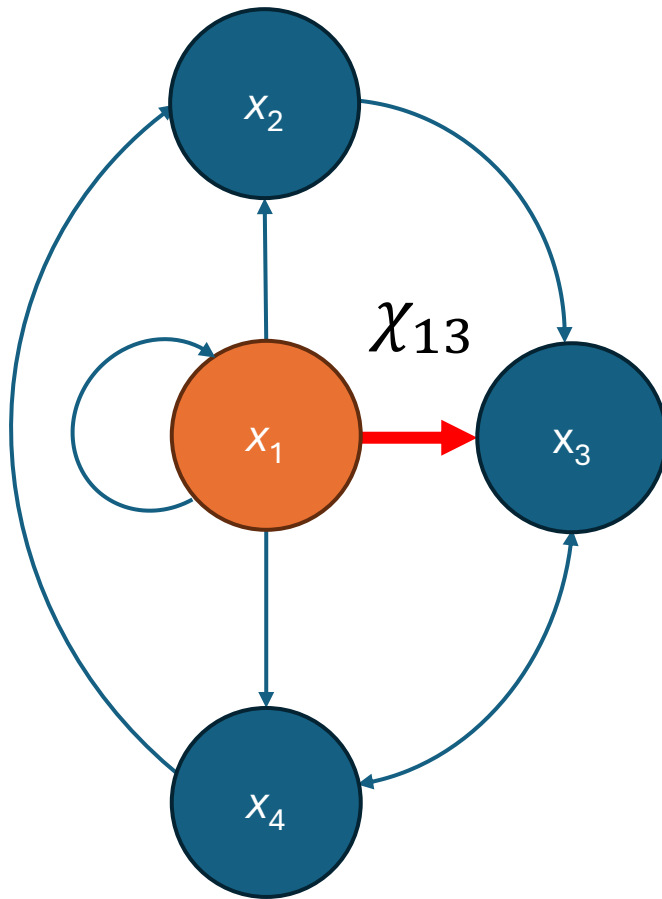
- The link event rate is then defined as: $\chi = s\chi^{\mathcal{F}}$

The Criticality of Orbital Regions



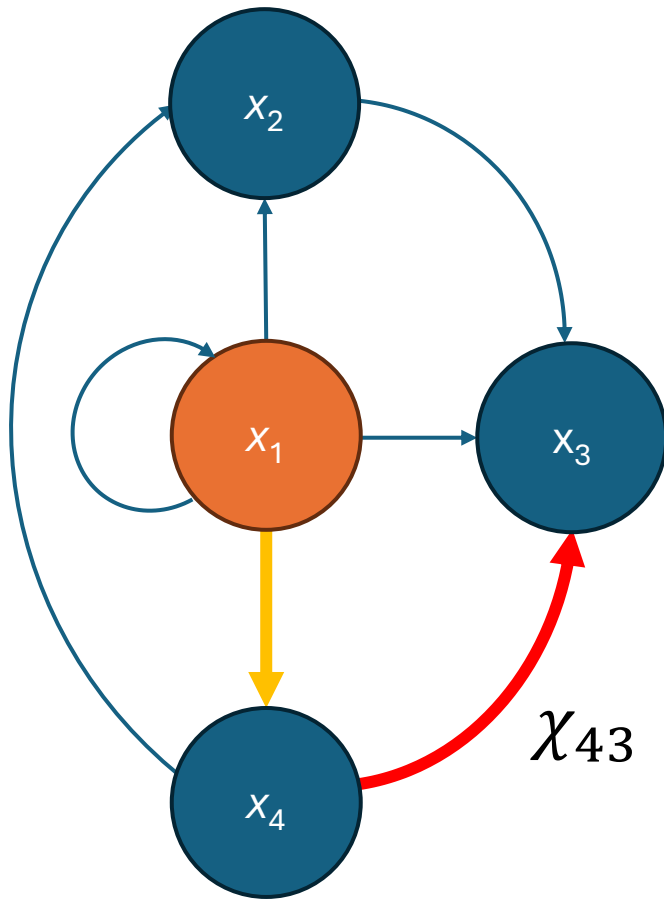
- Consider a path from region 1 to region 3 and say that χ_{23} is the link event rate of the weakest link

The Criticality of Orbital Regions



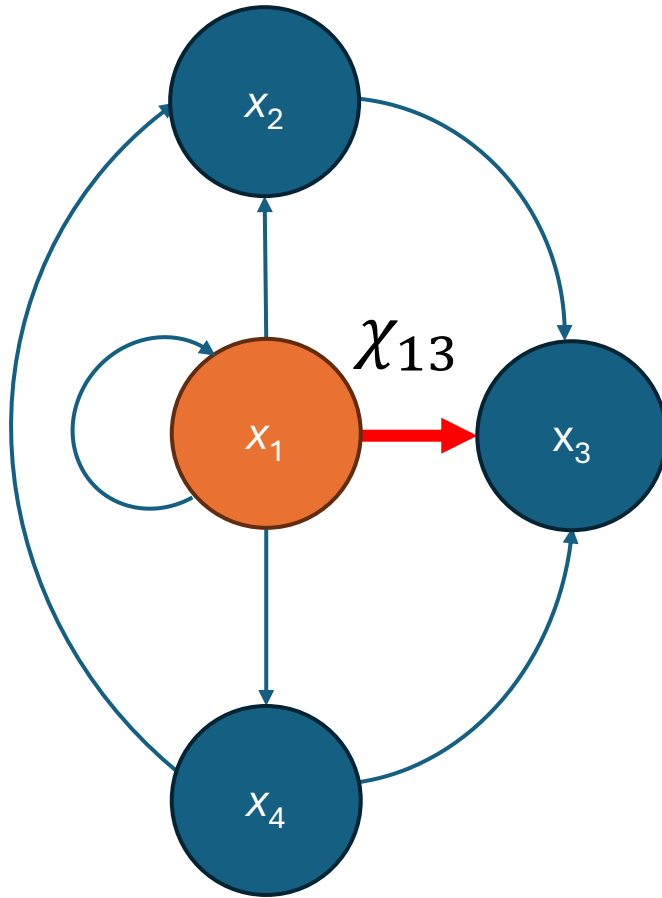
- Now consider a second path where χ_{13} is the link event rate of the weakest link

The Criticality of Orbital Regions



- And a third path which has now χ_{43} as is the link event rate of the weakest link

The Criticality of Orbital Regions



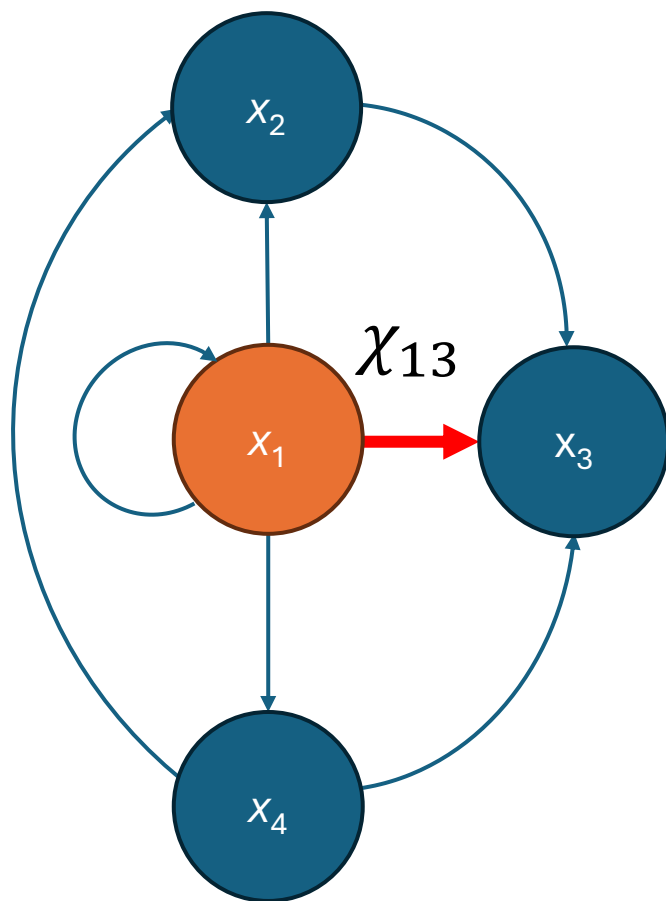
- If $\chi_{13} > \chi_{23} > \chi_{43}$ then 1 – 3 is a critical link.
- The sum of all critical links along all paths from a region o to all other n regions is the **criticality of region o** :

$$\alpha_o^{\chi} = \frac{1}{n} \sum_{i \wedge j \in I_{od}} n_{ij} \bar{\chi}_{ij}$$

where $\bar{\chi}_{ij}$ is the critical link along the paths from region o to region d and I_{od} are all the indexes of all the links along the paths from o to d

The weight n_{ij} is the *multiplicity* of the link or the number of times the link is critical along all the paths

The Criticality of Orbital Regions

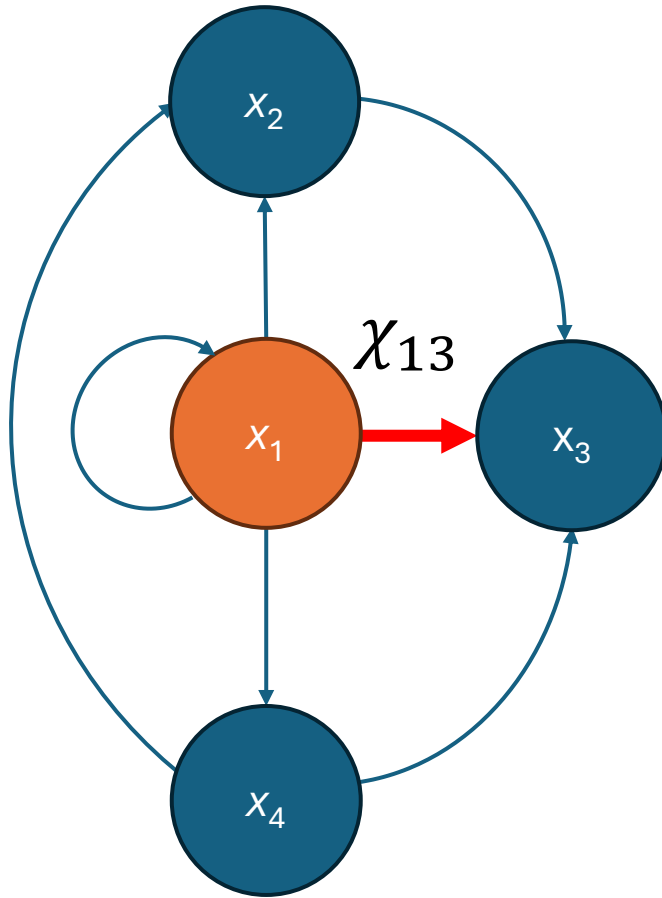


- The **criticality of an orbital region** can be understood as the weighted sum of all the event rates induced by that region and affecting the rest of the environment:

$$\alpha_o^x = \frac{1}{n} \sum_{i \wedge j \in I_{od}} n_{ij} \bar{\chi}_{ij}$$

- The criticality of the region measures how much that region is connected, or its **local connectivity**.

The Criticality of Orbital Regions



- We can also identify the most important link by computing the **link criticality score**:

$$\Theta_{ij} = \frac{n_{ij}}{n} \bar{\chi}_{ij}$$

- From which the **local connectivity** is:

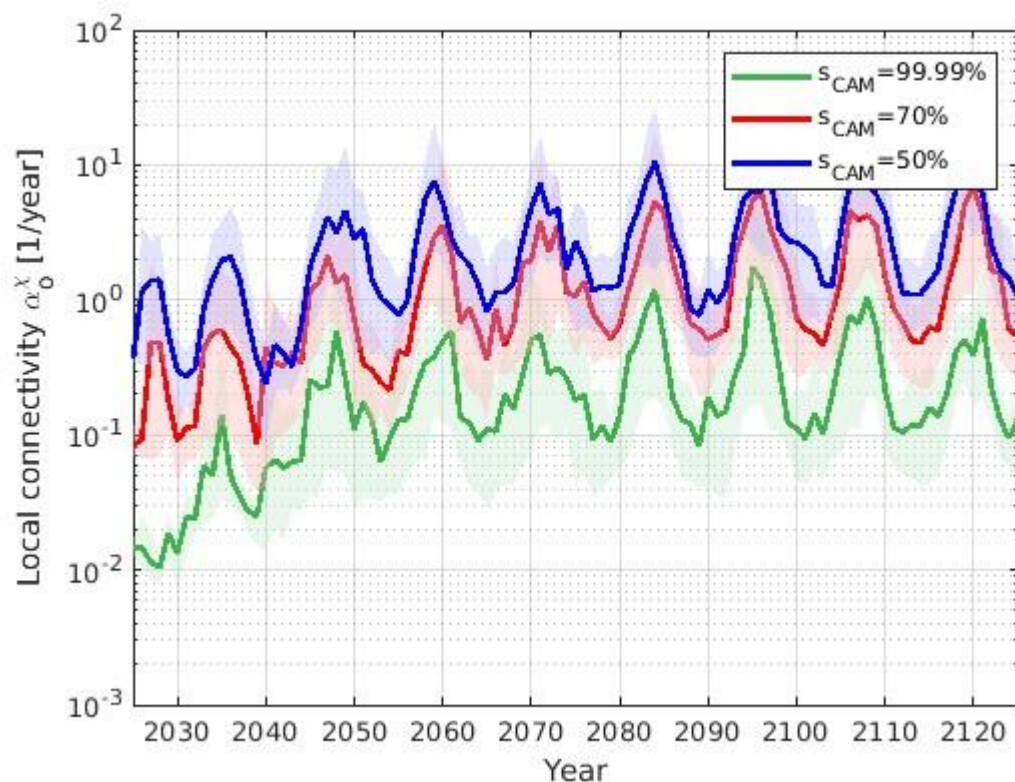
$$\alpha_o^{\chi} = \sum_{i \wedge j \in I_{od}} \Theta_{ij}$$

Example: Criticality of the Starlink Region

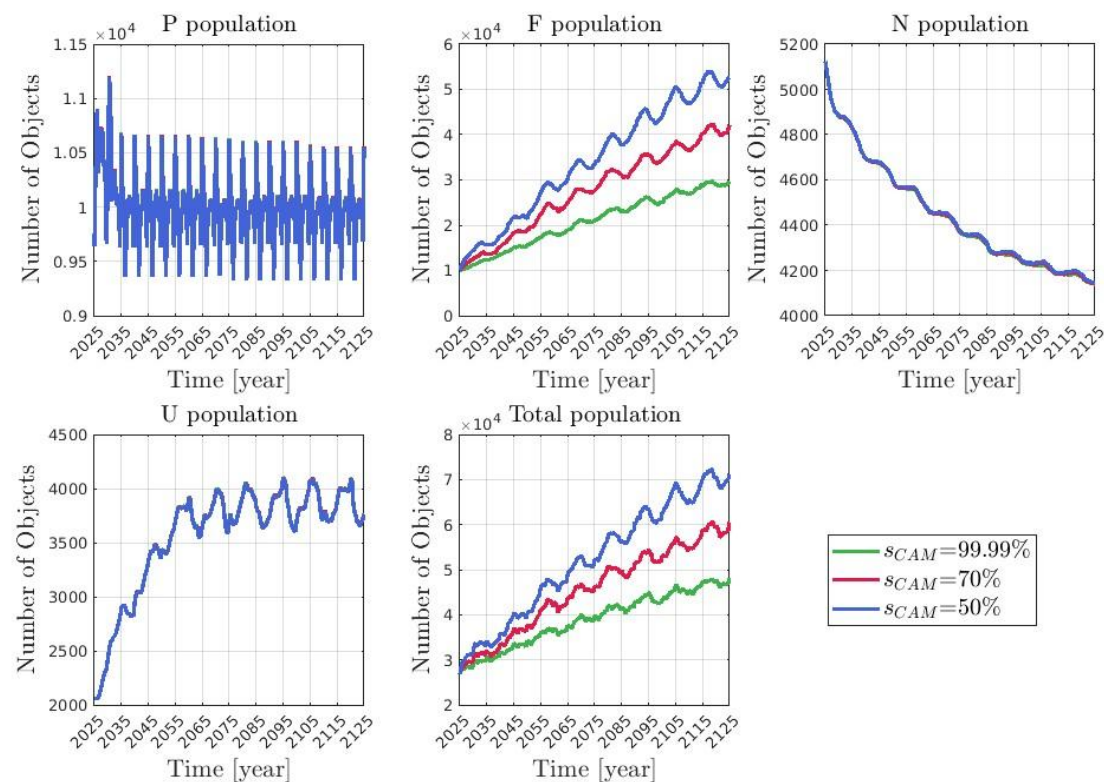
- Baseline case: propagating the initial IADC population as of January 2025 without future launches.
- As of January 2025, the Starlink constellation consists of 6,797 satellites.
- Three scenarios are considered
 - Baseline without Starlink
 - Baseline with Starlink, considering satellites replenishment
 - Baseline with Starlink, assuming all satellites are inactive

Impact of Starlink with Replenishment

- Criticality of Starlink region at 550km for different success rates of CAM.
- CAM success rate (s_{CAM}): 50%, 70%, 99.99%



Criticality of Starlink Region



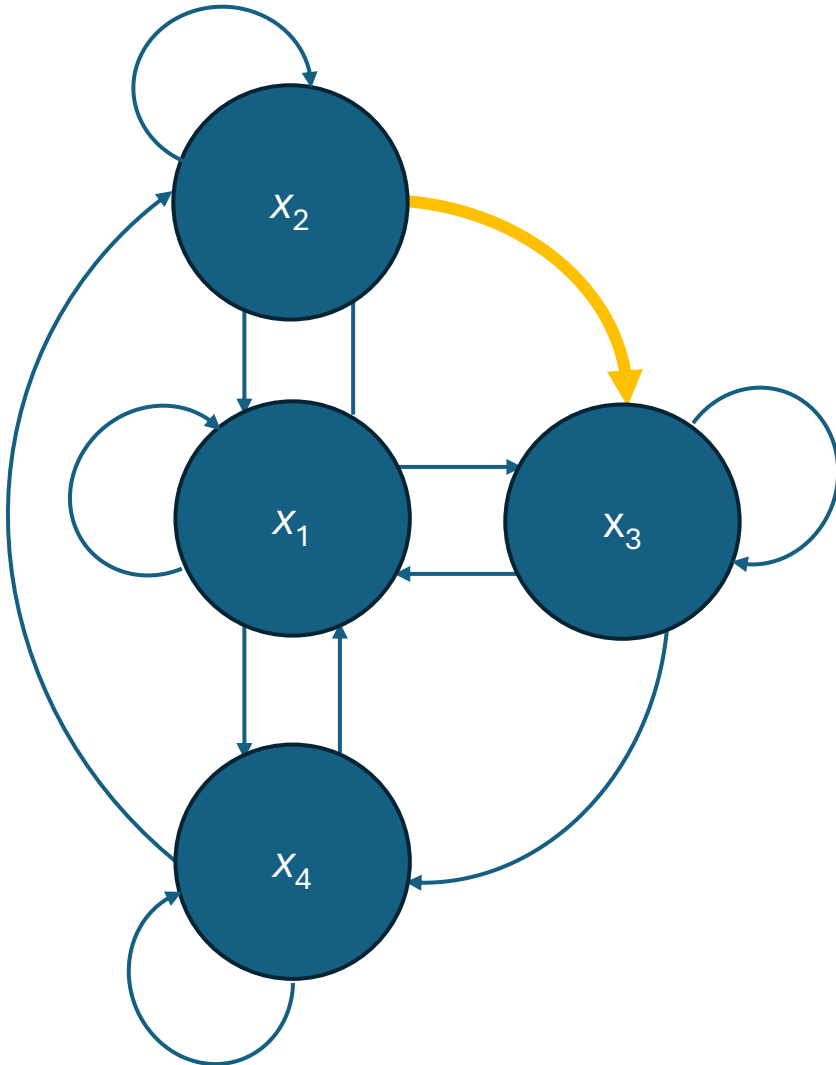
Mean Evolution of the Global Population

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Global and Local Connectivity



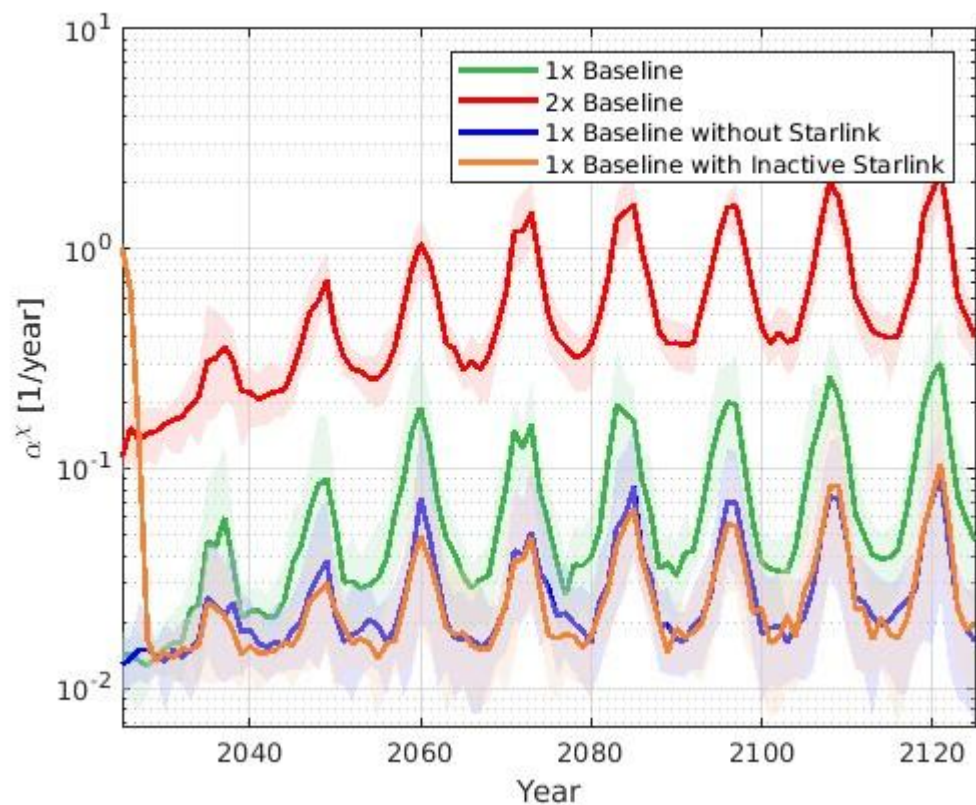
- The global connectivity of the whole environment is simply:

$$\alpha^x = \frac{1}{n^2} \sum_i^n \sum_j^n n_{ij} \bar{\chi}_{ij}$$

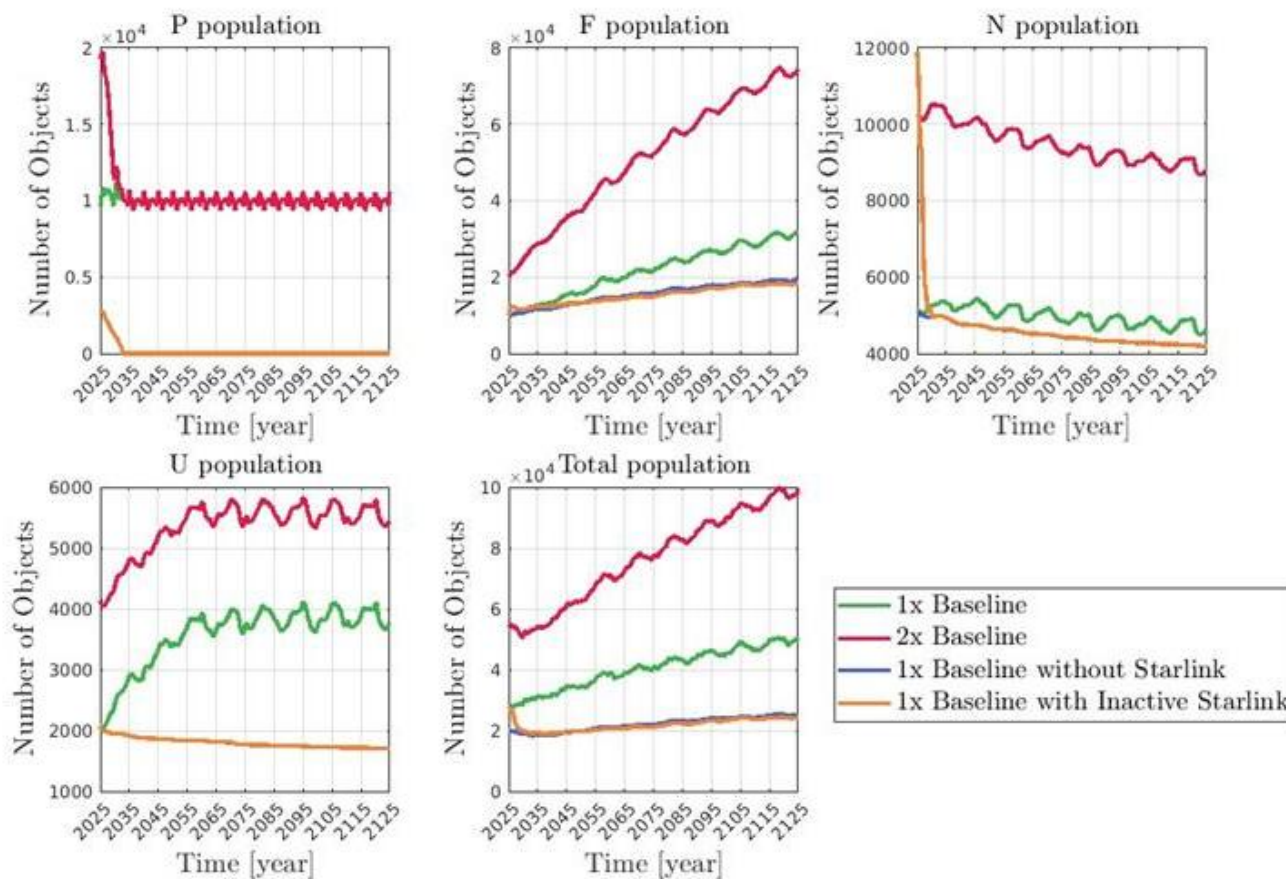
or the average sum of the criticalities of all the links.

The stronger the connectivity the more fragile the environment.

Global Connectivity



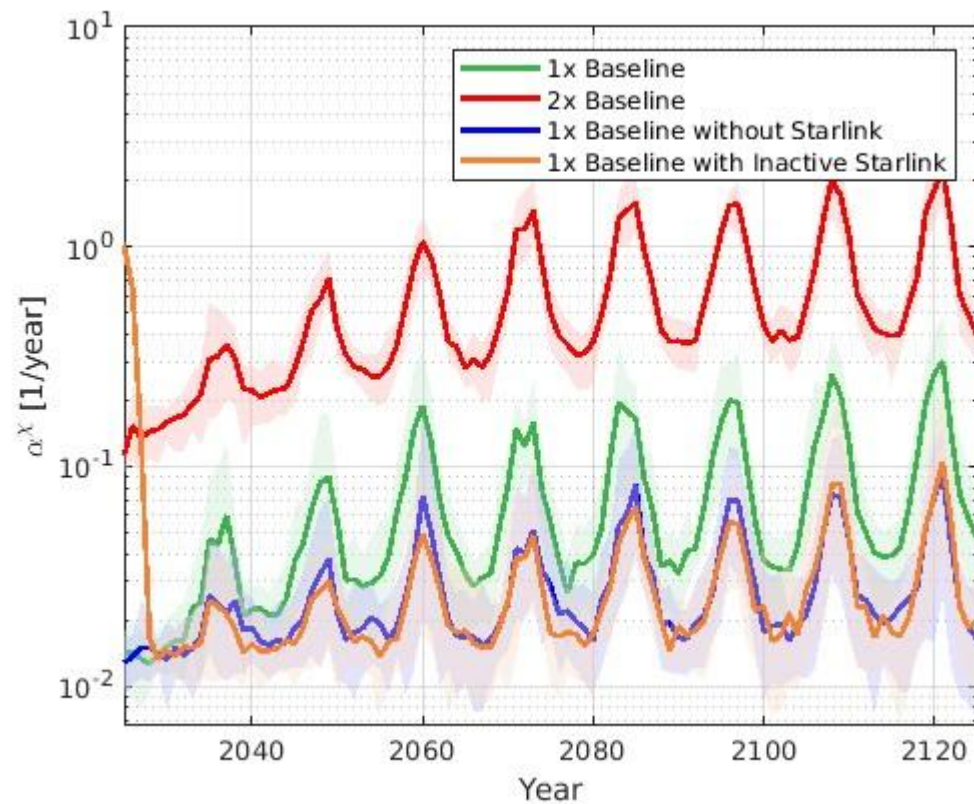
Global Connectivity



Mean Evolution of the Global Population

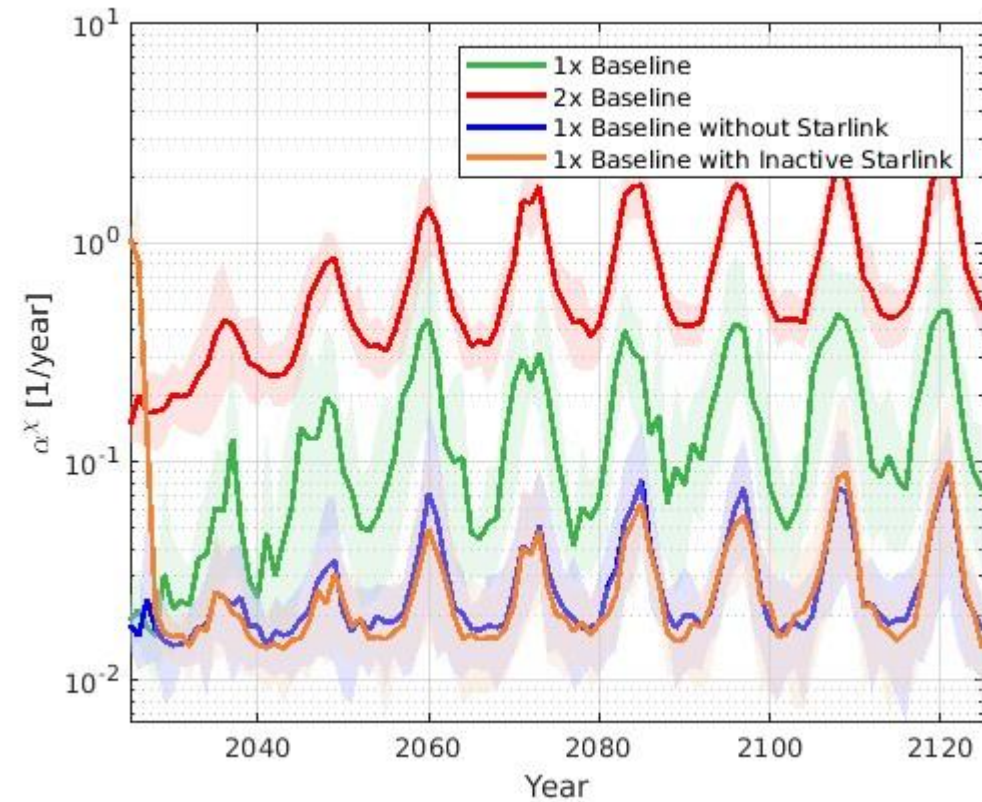
Global Connectivity

Without Lethal non-trackable



Global Connectivity

With Lethal non-trackable

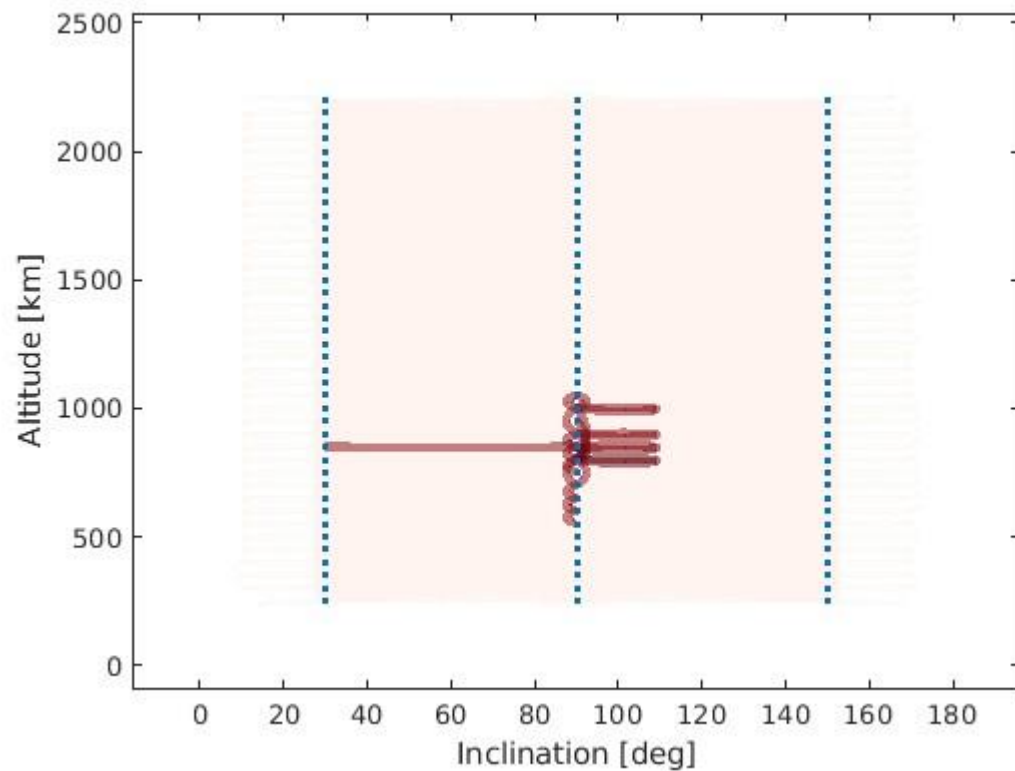


Global Connectivity

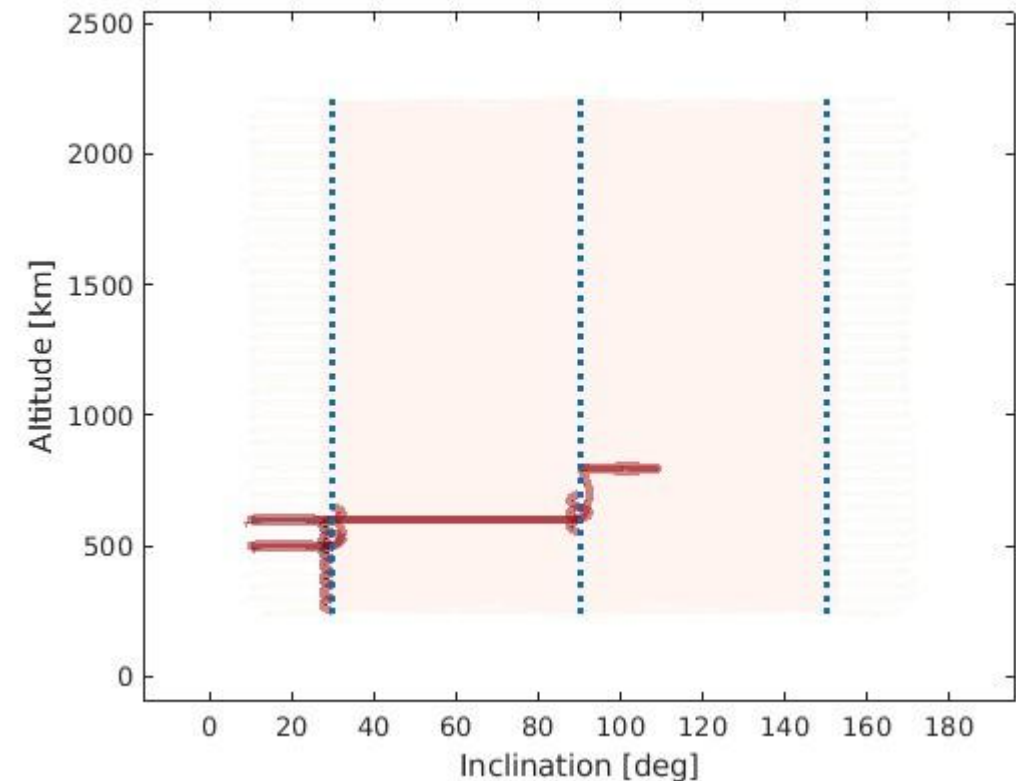
Impact of Starlink: top 20 most critical links

Links are ranked according to the criticality score: $\Theta_{ij} = \frac{n_{ij}}{n} \bar{\chi}_{ij}$

All Starlink satellites are active



All Starlink satellites are inactive



Final Remarks

- The critical number of objects in space is not sufficient to measure the status of the environment.
- Measuring the number of critical events is better quantification of how active and connected the environment is.
- A divergent connectivity dynamics implies an operational runaway effect.
- We can use the global connectivity as an alternative measure of the environment carrying capacity.
- An additional complementary metric quantifying the time at which events diffuse is potentially required to provide a complete picture.



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