Chemical module of Geant4-DNA

NEWS AND PERSPECTIVES

HIROSHIMA - 2015 GEANT4-DNA TUTORIAL

Water radiolysis ...

A REMINDER



Contribution to cell death of indirect effects VS LET



Modelling chemistry in Geant4 ?

PHYSICO-CHEMICAL & CHEMICAL STAGES









t=10⁻¹²s

t=10⁻¹⁵s

Physico-chemical stage

10

 $t = 10^{-6}s$

Electronic state of water molecule	Dissociation channels	Fraction (%)	
All single ionization states: H_2O^+	$H_3O^+ + OH$	100	
Excitation state A1B1:	•OH + H•	65	
$(1b1) \rightarrow (4a1/3s)$	$H_2O + \Delta E$	35	
Excitation state D1 A 1.	$H_3O^+ + OH + e_{aq}^-(AI)$	55	
$(2-1) \qquad (4-1/2-)$	$\cdot OH + \cdot OH + H_2$	15	
$(3a1) \rightarrow (4a1/3s)$	$H_2O + \Delta E$	30	
Excitation state:	$H_3O^+ + OH + e_{aq}^-(AI)$	50	
Rydberg, diffusion bands	$H_2O + \Delta E$	50	
Dissociative attachment: H ₂ O-	$\cdot OH + OH^{-} + H_2$	100	

- Situtation at 1 picosecond?
- Can be tuned by the user
- <u>Note:</u> the current version of the physico-chemistry is not compatible with the atomic deexcitation theory available in Geant4

Kreipl et al, 2009

t=10⁻¹²s

t=10⁻¹⁵s

Physico-chemical stage

	Hole hopping	Product 1	Product 2	Product 3
H ₃ O ⁺ + •OH	\vec{R} (2 <i>nm</i>) (charge transfer)	0*	$\vec{R}(0.8 nm)^*$	
$H_{3}O^{+} + OH^{+}e_{aq}$ (AI)	$\vec{R}(2 nm)$ (charge transfer)	0*	$\vec{R}(0.8 nm)^*$	
•ОН + Н•		-1/18 ×	17/18 ×	
	0	$\vec{R}(2.4 nm)$	<i>R</i> (2.4 <i>nm</i>)	—
Н ₂ + •OH + •OH		-2/18 ×	$16/18 \times \vec{R}(0.8 \ nm) + 1/2 \times$	$16/18 imes \vec{R}(0.8 \ nm)$ -
	0	\vec{R} (0.8 nm)	$\vec{R}(1.1 nm)$	$1/2 \times \vec{R}(1.1 nm)$
H_2 + •OH + OH-	0	$-2/18 imes \vec{R}$ (0.8 nm)	$16/18 \times \vec{R}(0.8 nm)$	$16/18 imes \vec{R}(0.8 \ nm)$ -
			$+1/2 \times \vec{R}(1.1 nm)$	$1/2 \times \vec{R}(1.1 nm)$

- Where to place the radiolytic products?
- Defined in G4DNAWaterDissociationDisplacer

Kreipl et al, 2009

t=10⁻⁶s





Physico-chemical stage

t=10⁻⁶s

Electron – water anion recombination

$e^- + H_2O^+ \rightarrow channel 1 / 2 / 3$

▶ 15-25 % of water anions recombine with electrons at room temperature \rightarrow non negligeable

Decrease the number of solvated electrons

Form H₂ product, stable species

This reaction might be strongly temperature dependent





Physico-chemical stage

14

 $t = 10^{-6} s$

Models for e- + H2O+ \rightarrow channel 1 / 2 / 3

- Model 2: « Molecular dynamics »-like treatment :
 - The electron is still tracked using the cross sections of DEA & vibrational/rotational excitation
 - But the sub excitation electrons also migrate in the potential generated by all the nearby holes and electrons in diffusion
 - Accounts for the decceleration of the electrons, and the effect of « crowdy » regions
 - Note to G4 developers Model 2 would require the physics models to work in the chemistry framework, feasible, but G4VEmProcess should not store locally « track-dependent attributes » (e.g. theNumberOfInteractionLengthLeft)



t=10⁻¹²s



Physico-chemical stage

Dominant species at the end of the physicochemical stage

 H_3O^+ , e^-_{aq} , °OH

16

Chemical stage



Computational complexity

Well adapted for few molecules and heterogenously distributed Assumption : molecules are homogenously distributed into one voxel. More adapted for large N



Computational complexity



20

Diffusion Smoluchowski dynamics

Brownian motion – the Langevin equation

21

A particle in a fluid is slowed down by a friction force:



► If its mass is weak → the particle undergoes multiple collisions with the medium :

$$m\ddot{x} = -\alpha\dot{x} + F_{ext}(x) + \underbrace{\psi(t)}_{\text{Random force}}$$

► This is the so-called Langevin equation → stochastic

Langevin Equation

$$\ddot{x} = -\frac{1}{m}\dot{x} + \frac{F_{ext}(x)}{m} + \frac{\psi(t)}{m}$$

$$\ddot{x} = -\gamma \dot{x} + \frac{F_{ext}(x)}{m} + \underbrace{\Gamma(t)}_{\text{Langevin force}}$$

Where $\gamma = \frac{1}{m}$ • <u>Hypothesis</u>

> $\langle \Gamma(t) \rangle = 0$ $\langle \Gamma(t_1) \cdot \Gamma(t_2) \rangle = q \cdot \delta(t_1 - t_2)$ Very weak mass + thermal equilibrium $\Rightarrow \gamma \dot{x} \gg \ddot{x}$

$$\gamma \dot{x} = \frac{F_{ext}(x)}{m} + \underbrace{\Gamma(t)}_{\text{Langevin force}}$$

Smoluchowski-Debye Equation

23

Stochastic differential equation

$$\dot{x} = rac{F_{ext}(x)}{m\gamma} + rac{\Gamma(t)}{\gamma}$$

The position is described by a density probability function p described by a Fokker-Planck equation

$$\frac{\partial p(x,t|x_0,t_0)}{\partial t} = \left(\frac{\partial}{\partial x^2} \left(\mathbf{P} \cdot \mathbf{p}\right) - \frac{\partial}{\partial x} \cdot \left(\frac{F_{ext}(x)}{m\gamma} \cdot \mathbf{p}\right)\right)$$

where
$$\mathbf{D} = \frac{q^2}{2\gamma}$$
 with $\langle \Gamma(t_1) \cdot \Gamma(t_2) \rangle = \mathbf{q} \cdot \delta(t_1 - t_2)$

Smoluchowski-Debye Equation

24

Solutions of the SDE in absence of external field

$$p(x,t|x_0,t_0) = \frac{1}{\left(4\pi \left(t-t_0\right)\right)^{\frac{1}{2}}} \exp\left(-\frac{(x-x_0)^2}{4\left(t-t_0\right)}\right) \qquad \text{in 1D}$$

$$p(r,t|r_{0},t_{0}) = \frac{4\pi r^{2}}{\left(4\pi D(t-t_{0})\right)^{\frac{3}{2}}} \exp\left(-\frac{(r-r_{0})^{2}}{4D(t-t_{0})}\right) \quad \text{in S}$$

where $P = \frac{q^{2}}{2\gamma}$ with $\langle \Gamma(t_{1}) \cdot \Gamma(t_{2}) \rangle = q \cdot \delta(t_{1}-t_{2})$

3D integrated over the angles

The simulation is a succession of time steps Δt where $\Delta t = t - t_0$ →

Illustration in Geant4-DNA





Diffusion-controlled reactions in Geant4-DNA, J Comp Phys (2014), 274, 841-882

Standard Geant4 transport VS Brownian motion



	Standard transport of Geant4	Brownian motion
Position and velocity	Deterministic (between two interactions points)	Stochastic
Path-volume Intersection	« Exact » intersection computable	Is expressed in terms of probability
Equation of motion	Newton	Fokker-Planck (stochastic equation of motion)

(Some) requirements for Brownian dynamics simulations

27

Given initial positions at time t=0 and Δt \rightarrow sample probable positions Time-driven stepping Δt known Δx ? Given initial position, distance to a boundary Problem known as the \rightarrow sample a time at which the particle can « first passage time » cross the boundary Space-driven stepping Δx known Δt **?** Given a time step Δt , initial and final Problem known as the boundary «Brownian bridge» \rightarrow what is the probability that the Brownian $\Delta x_0 \& \Delta x_f$ known for Δt crossed a boundary during the step **Cross boundary**?

(Some) requirements for Brownian dynamics simulations

Not yet used in the current version

Given initial positions at time I=0 and

At and knowing distances from

reflective boundaries \rightarrow sample probable positions for a selected time

step knowing that the particle will be

reflected by the surface

∆t known $x_0 \& x_f$?

Time-driven stepping with reflective boundaries

1/ Time-driven stepping



Time steps are fixed Space steps are computed



Step-by-step method – the Ermack McCammon algorithm

$$\Delta x_i = \sqrt{2D\Delta t} \cdot N(0,1) + \frac{D \cdot \Delta t}{k_B T} \cdot F_i(x_i^0)$$

with $\Delta t \gg \frac{mD}{k_BT}$, N(0,1) is a random number sample from a Gaussian distribution of mean 0 and standard deviation 1

Equivalent to the Smoluchowski dynamics

Sampling probability functions

1/Time-driven stepping – Probability functions



2/ First passage time (First-hitting-time)

1)

Follow the trajectories step-by-step



► To avoid small time steps $\Delta t \rightarrow$ select Δt_{max} , the biggest time steps possible during which it is guaranteed the boundary is not crossed

For instance : select Δt_{max} such as

 $P\left(\Delta x, \Delta t_{max}\right) = 95\%$

- Δt_{max} are dynamical computed Space steps are sampled using Δt_{max}

One can use minimum allowed time steps Δt_{min} to guarantee that the time steps don't become too small

- When the particle gets close to the boundary by a certain threshold (could be time or space based) use tricks to either
 - Adjust the final time step
 - Take the full time step Δt_{min} and use Brownian bridge to determine if the particle has crossed the boundary during Δt_{min}

2/ First passage time (First-hitting-time)



2) Sample the probability function to predict a first passage time

For instance, in 1D, invert in respect to time $P = \operatorname{erfc}\left(\frac{\Delta x}{2\sqrt{Dt}}\right)$

In 3D, the direct inversion technique does not work because the probability is more complicated

 Advantage compared to the step-by-step stepping : only one step taken (event-driven simulation)

3/ Brownian bridge

 $d_0 \& d_f$ known for Δt **Cross boundary ?**

Given a time step At, initial and final distances of a Brownian particle from a boundary -> what is the probability that the Brownian crossed a boundary during the step

$$P = exp\left(-\frac{d_0d_f}{D\cdot \Delta t}\right)$$

Where d_0 is the initial distance from the boundary and d_f is the final distance from the boundary

Note: The current implementation is a 1D approximation



Reactions SMOLUCHOWSKI REACTION MODEL – DIFFUSION CONTROLLED

Reaction rate constant 1/2 35

Chemical master equation (reaction level – well mixed chemical system)

Number of species : $\vec{N}(t) = (N_1(t), N_2(t), ..., N_n(t))$ Time evolution of $\vec{N}(t)$ is stochastic and defined by the density probability p

$$\frac{\partial p(\vec{N}(t),t)}{\partial t} = \sum_{r=\text{bimolecular reactions}} a_r(\vec{N}(t) - \vec{v_r}) \cdot p(\vec{N}(t) - \vec{v_r}, t) - \sum_{r=\text{unimolecular reactions}} a_r(\vec{N}(t)) \cdot p(\vec{N}(t), t)$$

Ordinary differential equations

$$\frac{d\langle N_k \rangle}{dt} = \sum_{i,j=\text{bimolecular reactions}} \pm k_{ij} \cdot \langle N_i \rangle \cdot \langle N_j \rangle + \sum_{i=\text{unimolecular reactions}} \pm k_i \cdot \langle N_i \rangle$$

Observed reaction rate constant

$$A + B \xrightarrow{k_{obs}} P$$

$$\frac{d\langle P\rangle}{dt} = \mathbf{k_{obs}}\langle A\rangle\langle B\rangle$$



Reaction rate constant 2/2

37

Reaction rate constant when complex life time very short

$$A + B \stackrel{k_C}{\underset{k_D}{\leftrightarrow}} (A:B) \stackrel{k_R}{\to} P$$

$$\frac{d\langle P \rangle}{dt} = \frac{k_R}{k_D + k_R} \left(k_C \cdot \langle A \rangle \langle B \rangle - \frac{d\langle C \rangle}{dt} \right)$$

If
$$\frac{d\langle C \rangle}{dt} \to 0$$
 since $\frac{d\langle P \rangle}{dt} = k_{obs} \langle A \rangle \langle B \rangle$

$$k_{obs} = \frac{k_R k_C}{k_D + k_R}$$

NB: this steady-state approximation will be removed with next versions of the chemistry

Reaction rate constant in case of diffusion-controlled reaction (also called diffusion-limited reaction)

$$\lim_{k_R \to \infty} k_{obs} = \lim_{k_R \to \infty} \frac{k_C}{\frac{k_D}{k_R} + 1} = k_C \to k_{obs} \approx k_C$$

Note: when k_R is finite, we fall in the case of the so-called partially diffusion-controlled reactions for which models are being considered for next versions of Geant4-DNA chemistry

Smoluchowski definition of ³⁸ reaction rate constant

Link between microscopic description and reaction rate constant :

k = flow of particles in solvation cage

$$\boldsymbol{k_{C}} = \mathcal{N}_{A} \cdot \boldsymbol{V} \cdot \int_{dS} \vec{j} \cdot \vec{dS}$$



In the absence of external field:

$$k_{C} = 4\pi \mathcal{N}_{A} \mathbf{D} R_{0}$$

Where D is the sum of diffusion coefficients and R_0 the sum of the radius of the cages

In case of Coulombic field

$$k_{C} = \frac{4\pi \mathcal{N}_{A} \mathbf{P} R_{C}}{\exp\left(\frac{R_{C}}{R_{0}}\right) - 1}$$

Using Smoluchowski theory, we can linked observed reaction rate constantss with reaction radius

With $R_C = \frac{q_1 q_2}{\epsilon_r \epsilon_0 k_B T}$ is the so-called Onsager radius

The implemented method

- STEP-BY-STEP WITH DYNAMICS TIME STEPS AND BROWNIAN BRIDGE
- DIFFUSION-CONTROLLED REACTIONS

The step -by-step method: principle

 $t = 10^{-6} s$

40

Yes

____r ≺-----

r < Reaction radius R ?

NO

Etape chimique

Step-by-step method

Interaction Can the molecules react ? Criterium: separation distance

2. Take one **diffusion** step for all molécules, return to **1**)

t=10<u>-12</u>s___

The step -by-step method: reaction

41



Reaction calculated after each step Δt ...

Step-by-step: method: How to choose Δt ?

Two solutions have been implemented in Geant4-DNA

- 1) Select an arbitrary time step
 - Example : A la PARTRAC*

Step Δt are predefined and evolved along the simulation

Time interval (s)	$\Delta t (ps)$
Until 1.0×10^{-11}	0.1
1.0×10^{-11} - 1.0×10^{-10}	1
1.0×10^{-10} - 1.0×10^{-9}	3
1.0×10^{-9} - 1.0×10^{-8}	10
Above 1.0×10^{-8}	100

*Kreipl et al, Radiat Environ Biophys, **48**, 11-20 (2009)

- 2) Compute it in respect to the next reaction*
 - Explanation ...

*Michalik et al., Radiation Research **149**, 224-236 (1998)

▶ How to compute a <u>At</u> in order not to miss reactions?

Position at to



• How to compute a Δt in order to avoid missing reactions?



45



• How to compute a Δt in order to avoid missing reactions?



47



48



49



50

How to compute a At in order to avoid missing reactions?



<u>« Dynamical time step » technique*</u>

*Michalik et al., Radiation Research **149**, 224-236 (1998)

Dynamical time-step – Protective time-space spheres

51



The dynamical time steps method can be seen as defining « protective time-space spheres »* where diffusion steps can be taken safely without reaction

*This is just an illustration



Multiple smaller and smaller steps before reacting (or not)



Drawback of the dynamical time steps

54

Multiple smaller and smaller steps

Solution: impose a minimum time step



Drawback of the dynamical time steps

- Multiple smaller and smaller steps
- Solution: impose a minimum time step
- Problem : may miss reactions



Drawback of the dynamical time steps

56

- Multiple smaller and smaller steps
- Solution: impose a minimum time step
- Problem : may miss reactions

Solution: compute a probability of encounter when threshold time steps are used

Ś

Brownian bridge (1D approximation)

Reaction → search for the closest neighbor

Brute-force method

Compare all distances between N reactants

- > Number of elementary operations $\approx N^2/2$
- Drawback: CPU







t=10 ⁻¹⁵ s t=10 ⁻¹² s Chemical stage:			t=10 ⁻⁶ s 60	
parameters		Reaction	Reaction rate	
Species	Diffusion coefficient D (10 ⁻⁹ m ² s ⁻¹)			(10 ¹⁰ M ⁻¹ S ⁻¹)
H ₃ O +	9.0	lles(• OH + $e_{aq}^{-} \rightarrow OH^{-}$	2.95
H● OH ⁻	7.0 5.0	Molecu	$H \bullet + e_{aq} + H_2O \rightarrow OH + H_2$	2.65
e⁻ _{aq}	4.9	uctl	$H_3O^+ + e_{aq}^- \rightarrow H^{\bullet} + H_2O$	2.11
H ₂	4.8	nstr	$H \bullet + \bullet OH \to H_2O$	1.44
H_2O_2	2.8	n Co	$H_2O_2 + e_{aq} \rightarrow OH^- + \bullet OH$	1.41
		$H^{\bullet} + H^{\bullet} \to H_2$	1.20	
We followed the set of parameters $e_{aq} + e_{aq} + 2H_2O \rightarrow H_2$		$e_{aq}^{-} + e_{aq}^{-} + 2H_2O \rightarrow 2OH^{-} + H_2$	0.50	

 $\bullet OH + \bullet OH \rightarrow H_2O_2$

We tollowed the set of parameters published by the authors of the PARTRAC software (Kreipl et al., REB 2009). However, these parameters can be modified by the user.

Kreipl et al, 2009

0.44

ConstructReactionTable()

In

Class hierarchy

\$G4INSTALL/source/processes/electromagnetic/dna





G4DNA chemistry limitations

62

Usage

- Add reactions → data needed
- Limited number of simulated molecules (simulation limited to small volumes)
- Still a prototype, refactoring needed
 → Bad interface, bugs etc ...
- > Don't horitate to report to us a
- → Don't hesitate to report to us any bug

Models

- Particle-continuum representation
- \rightarrow CPU and memory consuming
- \rightarrow Run on a cluster
- No reversible reactions
- Diffusion controlled reactions only
- Real dissociation scheme of water molecule?
- Working on new models to handle partially diffused-controlled reactions with reversible intermediate state

Notes related to the delivered code

The code which is delivered to you today is a beta release

- Known bugs
 - When beaming protons, solvated electrons may get solvated out of the volume of the electron originate, you'll see warnings thrown by G4Navigator
 - Radiolytic products could also be generated out of the original volume for similar reasons
- Don't use this code for production!
- With final 10.2 release, minor interface change will occur (definition of species, UI commands ...)

Perspectives





Heterogenous to homogeneous phase

(radiolytic products orignally located close to the deposited energy then diffuse)

As LET increases → deterministic

 Biochemical processes
 → high N molecules
 → often used compartment approaches

Radical-biomolecules reactions (such as proteins, lipids ...)? Need to go to higher time (DNA repair – protein recruitment)

Develop a hybrid method combining particle-based and compartment representations



Computational complexity

Merging representations

67



One volume may have both **particle-based** (to handle low number of species) and **compartment-based/well-stirred** (to handle high number of species) representations

Alpha testers

- Major developments will not be released in official Geant4 releases before 2017/2018 – The code delivered to you today is been largely rewritten -> we need to test it
- Alpha testers should be experimented Geant4 Users/Developers only
- Role
 - Suggest new features / propose new developments
 - Report bugs, test the code for your use case
- Drawbacks
 - Unstable versions
 - You'll not be allowed to published any results coming out the alpha releases, neither the methods being developed
- Alpha testing starting in 2016

Acknowledgments





Special thank you to the organising committee

Dr Takashi Sasaki, KEK, Japan

Dr Koichi Murakami, KEK, Japan

Dr Shogo Okada, KEK, Japan

Dr Sébastien Incerti, CNRS/IN2P3/Bordeaux U., France

Dr Giovanni Santin, ESA/ESTEC, The Netherlands

Dr Hoang Tran, TDT University, Ho Chi Minh, Vietnam

Dr Makoto Asai, SLAC, US



Thank you 🙂

http://geant4-dna.org

MHP: 119eal

Questions?

Diffusion-controlled reactions in Geant4-DNA, J Comp Phys (2014), 274, 841-882